UNIT – 1

QUADRATIC EQUATIONS

Unit Outlines

After studying this unit the students will be able to:

- Define quadratic equation.
- Solve a quadratic equation in one variable by factorization.
- Solve a quadratic equation in one variable by completing square.
- Derive quadratic formula by using method of completing square.
- Solve a quadratic equation by using quadratic formula.
- Solve the equations of the **type** $ax^4 + bx^2 + c = 0$ by reducing it to the quadratic form.
- Solve the equations of the **type** $a\,p(x) + \dfrac{b}{p(x)} = c$.
- Solve reciprocal equations of the **type** $a\left(x^2+\dfrac{1}{x^2}\right)+b\left(x+\dfrac{1}{x}\right)+c=0$.
- Solve exponential equations involving variables in exponents.
- Solve equations of the **type** $(x + a)(x + b)(x + c)(x + d) = k$ **where** $a + b = c + d$.
- Solve radical equations of the types

(i) $\sqrt{ax+b} = cx+d$

(ii) $\sqrt{x+a}+\sqrt{x+b} = \sqrt{x+c}$

(iii) $\sqrt{x^2+px+m}+\sqrt{x^2+px+n} = q$

DEFINITION

Quadratic Equation:

An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a **quadratic equation** or an equation of the **second degree.**

Standard form of Quadratic Equation: (in one variable)

$ax^2 + bx + c = 0$ where $a \neq 0$ and a, b, c are real numbers is called the **general** or **standard** form of quadratic equation. Here x is variable and "**a**" is co-efficient of x^2 "b" is co-efficient of x and "c" is constant term.

Pure quadratic Equation:

If b = 0 in quadratic equation $ax^2 + bx + c = 0$ then it is called a **pure equation** an equation of the form $ax^2 + c = 0$ is

e.g.
i. $25x^2 + 7 = 0$
ii. $3x^2 + 9 = 0$
iii. $6x^2 - 10 = 0$

Linear Equation:

If a = 0 in $ax^2 + bx + c = 0$ then it reduced to a linear equation $bx + c = 0$ in one variable "x".

e.g; $3x + 5 = 0$, $2x - 7 = 0$

Solution of Quadratic Equations:

To find solution of quadratic equation, following methods are used:

i. factorization ii. completing square
iii. quadratic formula

Solution of Factorization:

In this method write the quadratic equation in standard form:

$ax^2 + bx + c = 0$ ……………..(i)

If two numbers "r" and "s" can be found for equation (i) such that **r + s = b** and **rs = ac** then $ax^2 + bx + c = 0$ can be factorize into two linear factors.

EXERCISE 1.1

Q.1 Write the following quadratic equations in the standard form and point out pure quadratic equations.

(i) $(x + 7)(x - 3) = -7$

Sol: We have

$$
\begin{aligned}
x(x-3) + 7(x-3) &= -7\\
x^2 - 3x + 7x - 21 &= -7\\
x^2 + 4x - 21 &= -7\\
x^2 + 4x - 21 + 7 &= 0\\
x^2 + 4x - 14 &= 0
\end{aligned}
$$

So, given equation is in standard form.

(ii) $\dfrac{x^2+4}{3} - \dfrac{x}{7} = 1$

Sol:

$$
\begin{aligned}
\frac{7(x^2+4) - 3x}{21} &= 1\\
\frac{7x^2 + 28 - 3x}{21} &= 1\\
7x^2 + 28 - 3x &= 21\times 1\\
7x^2 - 3x + 28 - 21 &= 0\\
7x^2 - 3x + 7 &= 0
\end{aligned}
$$

Hence, equation is in standard form of quadratic equation.

(iii) $\mathbf{\frac{x}{x+1} + \frac{x+1}{x} = 6}$

Sol: We have

$\frac{x \cdot x + (x+1)(x+1)}{x(x+1)} = 6$

$\frac{x^2 + x(x+1) + 1(x+1)}{x(x+1)} = 6$

$\frac{x^2 + x^2 + x + x + 1}{x(x+1)} = 6$

$2x^2 + 2x + 1 = 6(x^2 + x)$

$2x^2 + 2x + 1 = 6x^2 + 6x$

$0 = 6x^2 - 2x^2 - 2x + 6x - 1$

$0 = 4x^2 + 4x - 1$ OR

$4x^2 + 4x - 1 = 0$

Hence, equation is in standard form of quadratic equation.

(iv) $\mathbf{\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0}$

Sol: $\frac{x(x+4) - (x-2)(x-2) + 4x(x-2)}{x(x-2)} = 0$

$x^2 + 4x - (x^2 - 2x - 2x + 4) + 4x^2 - 8x = 0\ [x(x-2)]$

$x^2 + 4x - x^2 + 2x + 2x - 4 + 4x^2 - 8x = 0$

$8x - 8x - 4 + 4x^2 = 0$

$4x^2 - 4 = 0$ in pure form.

$\mathbf{4x^2 - 4 = 0}$

$4(x^2 - 1) = 0$

$x^2 - 1 = \frac{0}{4}$

$x^2 - 1 = 0$

So, equation is pure quadratic equation.

(v) $\mathbf{\frac{x+3}{x+4} - \frac{x-5}{x} = 1}$

$\frac{x(x+3) - (x-5)(x+4)}{x(x+4)} = 1$

$x^2 + 3x - [x^2 - 5x + 4x - 20] = 1[x(x+4)]$

$x^2 + 3x - x^2 + 5x - 4x + 20 = x(x+4)$

$3x - 4x + 5x + 20 = x^2 + 4x$

$4x + 20 = x^2 + 4x$

$0 = x^2 + 4x - 4x - 20$

$0 = x^2 - 20$ OR

$x^2 - 20 = 0$

So this is pure quadratic equation.

(vi) $\mathbf{\frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}}$

Sol: $\frac{(x+1)(x+3) + (x+2)(x+2)}{(x+2)(x+3)} = \frac{25}{12}$

$\frac{x^2 + 3x + x + 3 + x^2 + 2x + 2x + 4}{x^2 + 3x + 2x + 6} = \frac{25}{12}$

$\frac{2x^2 + 8x + 7}{x^2 + 5x + 6} = \frac{25}{12}$

$12(2x^2 + 8x + 7) = 5(x^2 + 5x + 6)$

$24x^2 + 96x + 84 = 25x^2 + 125x + 150$

$25x^2 - 24x^2 + 125x - 96x + 150 - 84 = 0$

$x^2 + 29x + 66 = 0$ OR

Hence, equation is standard form of quadratic equation.

Q.2 Solve by Factorization.

(i) $\mathbf{x^2 - x - 20 = 0}$

Sol. $x^2 - 5x + 4x - 20 = 0$

$x(x-5) + 4(x-5) = 0$

$(x+4)(x-5) = 0$

By taking both values equal to zero

$x + 4 = 0$	$x - 5 = 0$
$x = -4$	$x = 5$

Solution set = $\{-4, 5\}$

(ii) $\mathbf{3y^2 = y(y-5)}$

Sol: $3y^2 = y^2 - 5y$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0 \quad y(2y+5) = 0$

By taking both values equal to zero

$Y = 0$	$2y + 5 = 0$
$Y = 0$	$2y = -5$
	$Y = \frac{-5}{2}$

Solution set $= \left\{0, \frac{-5}{2}\right\}$

(iii) $\mathbf{4 - 32x = 17x^2}$

Sol: $0 = 17x^2 + 32x - 4$

OR $17x^2 + 34x - 2x - 4 = 0$

$17x(x+2) - 2(x+2) = 0$

$(17x - 2)(x+2) = 0$

$(17x - 2)(x+2) = 0$

By taking both values equal to 0.

$17x - 2 = 0$	$x + 2 = 0$
$17x = 2$	$x = -2$
$x = \frac{2}{17}$	

Solution set = $\{-2, \frac{2}{17}\}$

(iv) $\mathbf{x^2 - 11x = 152}$

Sol: $x^2 - 11x - 152 = 0$

$x^2 - 19x + 8x - 152 = 0$

$x(x-19) + 8(x-19) = 0$

$(x+8)(x-19) = 0$

By taking both values equal to 0.

$x + 8 = 0 \quad | \quad x - 19 = 0$

$x = -8 \quad | \quad x = 19$

Solution set = {–8, 19}

(v) $\dfrac{x+1}{x} + \dfrac{x}{x+1} = \dfrac{25}{12}$

Sol: $\dfrac{x+1}{x} + \dfrac{x}{x+1} = \dfrac{25}{12}$

$$\frac{(x+1)(x+1) + x \cdot x}{x(x+1)} = \frac{25}{12}$$

$$\frac{x^2 + x + x + 1 + x^2}{x^2 + x} = +\frac{25}{12}$$

$$\frac{x^2 + 2x + 1 + x^2}{x^2 + x} = \frac{25}{12}$$

$$\frac{2x^2 + 2x + 1}{x^2 + x} = \frac{25}{12}$$

$12(2x^2 + 2x + 1) = 25(x^2 + x)$

$24x^2 + 24x + 12 = 25x^2 + 25x$

$25x^2 - 24x^2 + 25x - 24x - 12 = 0$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x + 4) - 3(x + 4) = 0$

$(x - 3)(x + 4) = 0$

By taking both values equal to 0.

$x - 3 = 0 \quad | \quad x + 4 = 0$

$x = 3 \quad | \quad x = -4$

Solution set = {–4,3}

(vi) $\dfrac{2}{x-9} = \dfrac{1}{x-3} - \dfrac{1}{x-4}$

Sol: $\dfrac{2}{x-9} = \dfrac{1(x-4) - 1(x-3)}{(x-3)(x-4)}$

$$\frac{2}{x-9} = \frac{x - 4 - x + 3}{x^2 - 3x - 4x + 12}$$

$$\frac{2}{x-9} = -\frac{1}{x^2 - 7x + 12}$$

$2(x^2 - 7x + 12) = -1(x - 9)$

$2x^2 - 14x + 24 = -x + 9$

$2x^2 - 14x + x + 24 - 9 = 0$

$2x^2 - 13x + 15 = 0$

$2x^2 - 10x - 3x + 15 = 0$

$2x(x - 5) - 3(x - 5) = 0$

$(2x - 3)(x - 5) = 0$

By taking both values equal to 0.

$2x - 3 = 0 \quad | \quad x - 5 = 0$

$2x = 3 \quad | \quad x = 5$

$x = \dfrac{3}{2}$

Solution set = $\left\{\dfrac{3}{2}, 5\right\}$

Q.3 Solve the following equations by completing square.

(i) $7x^2 + 2x - 1 = 0$

$7x^2 + 2x = 1$

Dividing both sides by "7"

$$\frac{7x^2}{7} + \frac{2}{7}x = \frac{1}{7}$$

$$x^2 + \frac{2}{7}x = \frac{1}{7}$$

Adding both sides $\left(\dfrac{1}{7}\right)^2$

$$x^2 + \frac{2}{7}x + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{7+1}{49}$$

$\because (a + b)^2 = a^2 + 2ab + b^2$

$$\left(x + \frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square root on both sides

$$\sqrt{\left(x + \frac{1}{7}\right)^2} = \sqrt{\frac{8}{49}}$$

$\because \sqrt{a^2} = a$ $x^2 = a$ $x = \pm a$

$$x + \frac{1}{7} = \pm \frac{2\sqrt{2}}{7}$$

$$x = -\frac{1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

Solution set = $\left\{\dfrac{-1 \pm 2\sqrt{2}}{7}\right\}$

(ii) $\mathbf{ax^2 + 4x - a = 0 \quad a \neq 0}$

Sol. $ax^2 + 4x = a$

Dividing both sides by "a"

$$\frac{ax^2}{a} + \frac{4}{a}x = \frac{a}{a}$$

$$x^2 + \frac{4}{a}x = 1$$

Adding both sides $\left(\dfrac{2}{a}\right)^2$

$$x^2 + \frac{4}{a}x + \left(\frac{2}{a}\right)^2 = 1 + \left(\frac{2}{a}\right)^2$$

$$x^2 + 2(x)\left(\frac{2}{a}\right) + \left(\frac{2}{a}\right)^2 = 1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2 = \frac{a^2+4}{a^2}$$

$\because$ $(a+b)^2 = a^2 + 2ab + b^2$

Taking square root on both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2} = \pm\sqrt{\frac{a^2+4}{a^2}} \quad \because \sqrt{a^2} = a$$

$$x+\frac{2}{a} = \pm\frac{\sqrt{a^2+4}}{a} \quad \because \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$x = -\frac{2}{a} \pm \frac{\sqrt{a^2+4}}{a}$$

$$x = \frac{-2\pm\sqrt{a^2+4}}{a}$$

Solution set $= \left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $11x^2 - 34x + 3 = 0$

Sol. $11x^2 - 34x = -3$

Dividing both sides by "11"

$$\frac{11x^2}{11} - \frac{34}{11}x = \frac{-3}{11}$$

$$x^2 - \frac{34}{11}x = \frac{-3}{11}$$

Adding both side by $\left(\frac{17}{11}\right)^2$

$$x^2 - \frac{34}{11}x + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \left(\frac{17}{11}\right)^2$$

$$x^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$\left(x-\frac{17}{11}\right)^2 = \frac{-33+289}{121}$$

$\because$ $(a-b)^2 = a^2 - 2ab + b^2$

$$\left(x-\frac{17}{11}\right)^2 = \frac{256}{121}$$

Taking square root on both sides

$$\left(x-\frac{17}{11}\right)^2 = \pm\sqrt{\frac{256}{121}}$$

$\because$ $\sqrt{a^2} = a$
$x^2 = a$
$x = \pm a$

$$x-\frac{17}{11} = \pm\frac{16}{11}$$

$x-\frac{17}{11} = \frac{16}{11}$	$x-\frac{17}{11} = \frac{-16}{11}$
$x = \frac{17}{11} + \frac{16}{11}$	$x = \frac{-16}{11} + \frac{17}{11}$
$x = \frac{17+16}{11}$	$x = \frac{-16+17}{11}$
$x = \frac{33}{11}$	$x = \frac{1}{11}$
$x = 3$	

Solution set $= \left\{3, \frac{1}{11}\right\}$

(iv) $\ell x^2 + mx + n = 0, \quad \ell \neq 0$

Sol. $\ell x^2 + mx = -n$

Dividing both sides by "ℓ"

$$\frac{\ell x^2}{\ell} + \frac{mx}{\ell} = \frac{-n}{\ell}$$

$$x^2 + \frac{mx}{\ell} = \frac{-n}{\ell}$$

Adding both side $\left(\frac{m}{2\ell}\right)^2$

$$x^2 + \frac{m}{\ell}x + \left(\frac{m}{\ell}\right)^2 = \frac{-n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$(x)^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = \frac{-n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

$\because$ $(a+b)^2 = a^2 + 2ab + b^2$

$$\left(x+\frac{m}{2\ell}\right)^2 = \frac{m^2-4\ell n}{4\ell^2}$$

$\because$ $\sqrt{a^2} = a, \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Taking square root on both sides

$$\sqrt{\left(x+\frac{m}{2\ell}\right)^2} = \pm\sqrt{\frac{m^2-4n\ell}{4\ell^2}}$$

$\because$ $\sqrt{a^2} = a$
$x^2 = a$
$x = \pm a$

$$x+\frac{m}{2\ell} = \pm\sqrt{\frac{m^2-4n\ell}{4\ell^2}}$$

$$x = \frac{-m}{2\ell} \pm \frac{\sqrt{m^2-4n\ell}}{2\ell}$$

$$x = \frac{-m\pm\sqrt{m^2-4n\ell}}{2\ell}$$

Solution set $= \left\{\frac{-m\pm\sqrt{m^2-4n\ell}}{2\ell}\right\}$

(v) $3x^2 + 7x = 0$

Sol. Dividing by "3" on both sides

$$\frac{3x^2}{3}+\frac{7}{3}x = \frac{0}{3}$$

$$x^2+\frac{7}{3}x = 0$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$$x^2+\frac{7}{3}x+\left(\frac{7}{6}\right)^2 = 0+\left(\frac{7}{6}\right)^2$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$$

Take square root on both sides

$$\sqrt{\left(x+\frac{7}{6}\right)^2} = \pm\sqrt{\left(\frac{7}{6}\right)^2}$$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$\because$ $(a+b)^2 = a^2+2ab+b^2$

$$x+\frac{7}{6} = \pm\frac{7}{6}$$

$x+\frac{7}{6} = \frac{7}{6}$	$x+\frac{7}{6} = -\frac{7}{6}$
$x = \frac{7}{6}-\frac{7}{6}$	$x = -\frac{7}{6}-\frac{7}{6}$
$x = \frac{7-7}{6}$	$x = \frac{-7-7}{6}$
$x = \frac{0}{6}$	$x = \frac{-14}{6}$
$x = 0$	$x = \frac{-7}{3}$

Solution set $= \left\{0, \frac{-7}{3}\right\}$

(vi) $\mathbf{x^2-2x-195=0}$

$x^2-2x = 195$

Adding on both side "$(1)^2$"

$x^2-2(x)(1)+(1)^2 = 195+(1)^2$

$(x-1)^2 = 195+1$

$\because$ $(a-b)^2 = a^2-2ab+b^2$

$(x-1)^2 = 196$

Taking square root on both sides

$\sqrt{(x-1)^2} = \pm\sqrt{196}$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$x-1 = \pm 14$

$x-1 = 14$	$x-1 = -14$
$x = 14+1$	$x = -14+1$
$x = 15$	$x = -13$

Solution set $= \{15, -13\}$

(vii) $\mathbf{-x^2+\frac{15}{2} = \frac{7}{2}x}$

Sol. $\frac{15}{2} = x^2+\frac{7}{2}x$

OR $x^2+\frac{7}{2}x = \frac{15}{2}$

Adding both side $\left(\frac{7}{4}\right)^2$

$$x^2+\frac{7}{2}x+\left(\frac{7}{4}\right)^2 = \frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$x^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2 = \frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2 = \frac{120+49}{16}$$ $\because (a+b)^2 = a^2+2ab+b^2$

$$\left(x+\frac{7}{4}\right)^2 = \frac{169}{16}$$

Take square root on both sides

$$\sqrt{\left(x+\frac{7}{4}\right)^2} = \pm\sqrt{\frac{169}{16}}$$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$$x+\frac{7}{4} = \pm\frac{13}{4}$$

$x+\frac{7}{4} = \frac{13}{4}$	$x+\frac{7}{4} = -\frac{13}{4}$
$x = \frac{13}{4}-\frac{7}{4}$	$x = -\frac{13}{4}-\frac{7}{4}$
$x = \frac{13-7}{4}$	$x = \frac{-13-7}{4}$
$x = \frac{6}{4}$	$x = \frac{-20}{4}$
$x = \frac{3}{2}$	$x = -5$

Solution set $= \left\{-5, \frac{3}{2}\right\}$

(viii) $\mathbf{x^2+17x+\frac{33}{4} = 0}$

Sol. $x^2+17x = \frac{33}{4}$

Adding both side $\left(\frac{17}{2}\right)^2$

$$x^2+17x+\left(\frac{17}{2}\right)^2 = \frac{-33}{4}+\left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$

$\because \quad (a + b)^2 = a^2 + 2ab + b^2$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33 + 289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = 64$$

Take square root on both sides, we get

$$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{64}$$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$$x + \frac{17}{2} = \pm 8$$

$x + \frac{17}{2} = 8$	$x + \frac{17}{2} = -8$
$x = 8 - \frac{17}{2}$	$x = -8 - \frac{17}{2}$
$= \frac{16 - 17}{2}$	$= \frac{-16 - 17}{2}$
$x = -\frac{1}{2}$	$x = -\frac{33}{2}$

Solution set $= \left\{\frac{-1}{2}, \frac{-33}{2}\right\}$

(ix) $\mathbf{4 - \frac{8}{3x + 1} = \frac{3x^2 + 5}{3x + 1}}$

Sol. $\frac{4(3x + 1) - 8}{(3x + 1)} = \frac{3x^2 + 5}{3x + 1}$

$12x + 4 - 8 = \left(\frac{3x^2 + 5}{3x + 1}\right)(3x + 1)$

$12x - 4 = 3x^2 + 5$

$0 = 3x^2 - 12x + 4 + 5$

$0 = 3x^2 - 12x + 9$

OR $3x^2 - 12x = -9$

Dividing both side by "3"

$\frac{3x^2}{3} - \frac{12x}{3} = \frac{-9}{3}$

$x^2 - 4x = -3$

Adding both side $(2)^2$

$(x)^2 - 4x + (2)^2 = -3 + (2)^2$

$(x)^2 - 4x + (2)^2 = -3 + 4$

$(x - 2)^2 = 1$

$\because \quad (a - b)^2 = a^2 - 2ab + b^2$

Taking square root on both sides

$\sqrt{(x - 2)^2} = \pm\sqrt{1}$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$x - 2 = \pm 1$

$x - 2 = 1$	$x - 2 = -1$
$x = 1 + 2$	$x = -1 + 2$
$x = 3$	$x = 1$

Solution set $= \{1,3\}$

(x) $\mathbf{7(x + 2a)^2 + 3a^2 = 5a(7x + 23a)}$

Sol. $7\{(x^2) + (2a)^2 + 2(x)(2a)\} + 3a^2 = 35ax + 115a^2$

$7(x^2 + 4a^2 + 4ax) + 3a^2 - 35ax - 115a^2 = 0$

$7x^2 + 28a^2 + 28ax + 3a^2 - 35ax - 115a^2 = 0$

$7x^2 - 7ax - 84a^2 = 0$

$7x^2 - 7ax = 84a^2$

Dividing both sides by "7"

$\frac{7x^2}{7} - \frac{7ax}{7} = \frac{84a^2}{7}$

$x^2 - ax = 12a^2$

$x^2 - ax = 12a^2$

Adding both side $\left(\frac{a}{2}\right)^2$

$$x^2 - ax + \left(\frac{a}{2}\right)^2 = 12a^2 + \left(\frac{a}{2}\right)^2$$

$$x^2 - 2(x)\left(\frac{a}{2}\right) + \left(\frac{a}{2}\right)^2 = 12a^2 + \frac{a^2}{4}$$

$$\left(x - \frac{a}{2}\right)^2 = \frac{48a^2 + a^2}{4}$$

$\because \quad (a - b)^2 = a^2 - 2ab + b^2$

$$\left(x - \frac{a}{2}\right)^2 = \frac{49a^2}{4}$$

Taking square root on both side

$$\sqrt{\left(x - \frac{a}{2}\right)^2} = \pm\sqrt{\frac{49a^2}{4}}$$

$\because \sqrt{a^2} = a$, $x^2 = a$, $x = \pm a$

$$x - \frac{a}{2} = \pm\frac{7a}{2}$$

$x - \frac{a}{2} = \frac{7a}{2}$	$x - \frac{a}{2} = \frac{-7a}{2}$
$x = \frac{7a}{2} + \frac{a}{2}$	$x = \frac{-7a}{2} + \frac{a}{2}$
$x = \frac{7a + a}{2}$	$x = \frac{-7a + a}{2}$

$x = \frac{8a}{2}$ | $x = \frac{-6a}{2}$

$x = 4a$ | $x = -3a$

Solution set $= \{4a, -3a\}$

QUADRATIC FORMULA

Derivation of Quadratic Formula by using Completing Square Method

The quadratic equation in standard form

$ax^2 + bx + c = 0, \qquad a \neq 0$

Dividing each term of equation by "a"

$\frac{a}{a}x^2 + \frac{b}{a}x = \frac{-c}{a}$

We get,

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Shifting constant term $\frac{c}{a}$ to the right we have,

$x^2 + \frac{b}{a}x = \frac{-c}{a}$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides, we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x^2 + 2\left(\frac{b}{2a}\right)(x) + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$\because \quad (a + b)^2 = a^2 + 2ab + b^2$

Taking square root on both sides

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$\because \sqrt{a^2} = a$

$x^2 = a$

$x = \pm a$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$ is known as quadratic formula.

EXERCISE 1.2

Q.1 Solve the following equations by using quadratic formula.

(i) $2 - x^2 = 7x$

Sol. $2 = x^2 + 7x$ OR

$x^2 + 7x - 2 = 0$ is in standard form.

Compare it with

$ax^2 + bx + c = 0$

$a = 1, \quad b = 7, \quad c = -2$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2+-4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set $= \left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 5, \quad b = 8, \quad c = 1$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2-4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64-20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm \left(\sqrt{4} \times \sqrt{11}\right)}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{2(-4 \pm \sqrt{11})}{10} \qquad x = \frac{(-4 \pm \sqrt{11})}{5}$$

Solution set $= \left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Sol. $\sqrt{3}x^2 + x - 4\sqrt{3} = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = \sqrt{3}$, $b = 1$, $c = -4\sqrt{3}$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(\sqrt{3}.\sqrt{3})}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(\sqrt{3^2})}}{2.\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2.\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2.\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2.\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2.\sqrt{3}}$$

$x = \frac{-1+7}{2.\sqrt{3}}$	$x = \frac{-1-7}{2.\sqrt{3}}$
$x = \frac{6}{2.\sqrt{3}}$	$x = \frac{-8}{2.\sqrt{3}}$
$x = \frac{3}{\sqrt{3}}$	$x = \frac{-4}{\sqrt{3}}$
Multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$	
$x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	
$x = \frac{3\times\sqrt{3}}{(\sqrt{3})^2}$	$x = \frac{-4}{\sqrt{3}}$
$x = \frac{3\times\sqrt{3}}{3}$	
$x = \sqrt{3}$	$x = \frac{-4}{\sqrt{3}}$

Solution set $= \left\{\sqrt{3}, \frac{-4}{\sqrt{3}}\right\}$

(iv) $4x^2 - 14 = 3x$

Sol. $4x^2 - 14 - 3x = 0$

$4x^2 - 3x - 14 = 0$

Compare with quadratic equation

$ax^2 + bx + c = 0$

$a = 4$, $b = -3$, $c = -14$

using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

Solution set $= \left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $6x^2 - 3 - 7x = 0$

Sol. $6x^2 - 7x - 3 = 0$

Compare it with quadratic equation

$ax^2 + bx + c = 0$ is in standard form.

$a = 6$, $b = -7$, $c = -3$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - (4)(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$x = \frac{7+11}{12}$	$x = \frac{7-11}{12}$
$x = \frac{18}{12}$	$x = \frac{-4}{12}$
$x = \frac{3}{2}$	$x = \frac{-1}{3}$

Solution set $= \left\{\frac{3}{2}, \frac{-1}{3}\right\}$

(vi) $3x^2 + 8x + 2 = 0$

Sol. Compare it with

$ax^2 + bx + c = 0$

$a = 3$, $b = 8$, $c = 2$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6} = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{10 \times 4}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{2(-4 \pm \sqrt{10})}{6}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Solution set $= \left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6} - \frac{4}{x-5} = 1$

Sol. $\frac{3(x-5) - 4(x-6)}{(x-6)(x-5)} = 1$

$$\frac{3x - 15 - 4x + 24}{x^2 - 6x - 5x + 30} = 1$$

$$\frac{-x + 9}{x^2 - 11x + 30} = 1$$

$-x + 9 = 1(x^2 - 11x + 30)$

$x^2 - 11x + 30 + x - 9 = 0$

$x^2 - 10x + 21 = 0$ in standard form

Compare it with

$ax^2 + bx + c = 0$

$a = 1, \quad b = -10, \quad c = 21$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{10 \pm 4}{2}$$

$x = \frac{10 + 4}{2}$ | $x = \frac{10 - 4}{2}$

$x = \frac{14}{2}$ | $x = \frac{6}{2}$

$x = 7$ | $x = 3$

Solution set $= \{7, 3\}$

(viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

Sol. $\frac{2x(x+2) - (4-x)(x-1)}{2x(x-1)} = \frac{7}{3}$

$$\frac{2x^2 + 4x - (4x - 4 - x^2 + x)}{2x(x-1)} = \frac{7}{3}$$

$$\frac{2x^2 + 4x - 4x + 4 + x^2 - x}{2x(x-1)} = \frac{7}{3}$$

$$\frac{2x^2 + x^2 + 4x - 4x - x + 4}{2x(x-1)} = \frac{7}{3}$$

$$\frac{3x^2 - x + 4}{2x^2 - 2x} = \frac{7}{3}$$

$3(3x^2 - x + 4) = 7(2x^2 - 2x)$

$9x^2 - 3x + 12 = 14x^2 - 14x$

$14x^2 - 14x - 9x^2 + 3x - 12 = 0$

$5x^2 - 11x - 12 = 0$

$5x^2 - 11x - 12 = 0$ is in standard form.

Compare it with

$ax^2 + bx + c = 0$

$a = 5, \quad b = -11, \quad c = -12$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-12)}}{2(5)}$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10} = \frac{11 \pm 19}{10}$$

$$x = \frac{11 + 19}{10}, \quad \frac{11 - 19}{10}$$

$$x = \frac{30}{10}, \quad \frac{-8}{10}$$

$$x = 3, \quad \frac{-4}{5}$$

Solution set $= \left\{\frac{-4}{5}, 3\right\}$

(ix) $\frac{a}{x-b} + \frac{b}{x-a} = 2$

Sol. $\frac{a(x-a) + b(x-b)}{(x-b)(x-a)} = 2$

$$\frac{ax - a^2 + bx - b^2}{x^2 - ax - bx + ab} = 2$$

$ax + bx - a^2 - b^2 = 2(x^2 - ax - bx + ab)$

$2x^2 - 2ax - 2bx + 2ab - ax - bx + a^2 + b^2 = 0$

$2x^2 - 2ax - ax - 2bx - bx + a^2 + b^2 + 2ab = 0$

$2x^2 - 3ax - 3bx + a^2 + b^2 + 2ab = 0$

$2x^2 - 3(a + b)x + (a + b)^2 = 0$ is in standard form.

Compare it with

$ax^2 + bx + c = 0$

Here $a = 2,\ b = -3(a + b),\ c = (a + b)^2$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3(a + b)) \pm \sqrt{(-3(a + b))^2 - 4(2)(a + b)^2}}{2(2)}$$

$$x = \frac{3(a + b) \pm \sqrt{9(a + b)^2 - 8(a + b)^2}}{4}$$

$$x = \frac{3(a + b) \pm \sqrt{(a + b)^2 (9 - 8)}}{4}$$

$$x = \frac{3(a + b) \pm \sqrt{(a + b)^2 (1)}}{4}$$

$$x = \frac{3(a + b) \pm (a + b)}{4}$$

$x = \frac{3(a + b) + a + b}{4}$	$x = \frac{3(a + b) - a - b}{4}$
$x = \frac{3a + 3b + a + b}{4}$	$x = \frac{3a + 3b - a - b}{4}$
$x = \frac{4a + 4b}{4}$	$x = \frac{2a + 2b}{4}$
$x = \frac{4(a + b)}{4}$	$x = \frac{2(a + b)}{4}$
$x = a + b$	$x = \frac{1}{2}(a + b)$

Solution set $= \left\{\frac{1}{2}(a + b), (a + b)\right\}$

(x) $-(\ell + m) - \ell x^2 + (2\ell + m)x = 0,\ \ell \neq 0$

Sol. $-(\ell + m) - \ell x^2 + (2\ell + m)x = 0$ **OR**

$\ell x^2 - (2\ell + m)x + (\ell + m) = 0$

is in standard form

Compare it with

$ax^2 + bx + c = 0$

$a = \ell,\quad b = -(2\ell + m),\ c = (\ell + m)$

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{-[-(2\ell + m)] \pm \sqrt{(-(2\ell\ m)^2 - 4(\ell)(\ell + m)}}{2(\ell)}$$

$$x = \frac{(2\ell + m) \pm \sqrt{(2\ell + m)^2 - 4\ell(\ell + m)}}{2(\ell)}$$

$$x = \frac{(2\ell + m) \pm \sqrt{4\ell^2 + m^2 + 4\ell m - 4\ell^2 - 4\ell m}}{2\ell}$$

$$x = \frac{(2\ell + m) \pm \sqrt{m^2}}{2\ell}$$

$$x = \frac{2\ell + m \pm m}{2\ell}$$

$x = \frac{2\ell + m + m}{2\ell}$	$x = \frac{2\ell + m - m}{2\ell}$
$x = \frac{2\ell + 2m}{2\ell}$	
$x = \frac{2(\ell + m)}{2\ell}$	$x = \frac{2\ell}{2\ell}$
$x = \frac{(\ell + m)}{\ell}$	$x = 1$

Solution Set $= \{\frac{(\ell + m)}{\ell}, 1\}$

Use of Quadratic Formula:

The quadratic formula is useful tool for solving all those equations which can or can not be factorized.

EXERCISE 1.3

Equation Reducible to Quadratic form:

Type (i) $ax^4 + bx^2 + c = 0$

Replacing $x^2 = y$ and $x^4 = y^2$ in equation

$ax^4 + bx^2 + c = 0$

We get a quadratic equation y

$ay^2 + by + c = 0$

Note: Solve this equations by any method

i. Factorization ii. Completing Square

iii. Quadratic Formula

Q.1 Solve the following equations.

Sol. $2x^4 - 11x^2 + 5 = 0$ ……..(i)

Let $x^2 = y$ ……..(ii)

$(x^2)^2 = y^2$

$x^4 = y^2$ ……..(iii)

Putting values in equation (i)

$2(y)^2 - 11(y) + 5 = 0$ is quadratic equation

$2y^2 - 10y - y + 5 = 0$

$2y(y - 5) - 1(y - 5) = 0$

$(2y - 1)(y - 5) = 0$

$2y - 1 = 0$	$y - 5 = 0$
$2y = 1$	$y = 5$
$y = \frac{1}{2}$	

From (ii)

$y = x^2$

$x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

From (ii)

$y = x^2$

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

Solution set $=\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\}$

Q.2. $2x^4 = 9x^2 - 4$

Sol. $2x^4 - 9x^2 + 4 = 0$ ……..(i)

Let $(x^2) = y$ ……..(ii)

$(x^2)^2 = y^2$

$(x^4) = y^2$ ……..(iii)

Putting the values in equation (i)

$2y^2 - 9y + 4 = 0$ is quadratic equation

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(2y-1)(y-4) = 0$

$2y - 1 = 0$

$2y = 1$

$y = \frac{1}{2}$

$y - 4 = 0$

$y = 4$

From (ii)

$x^2 = y$

$x^2 = \frac{1}{2}$

From (ii)

$x^2 = y$

$x^2 = 4$

Taking square root on both sides

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Solution set $= \{\pm\frac{1}{\sqrt{2}}, \pm 2\}$

Q.3. $5x^{1/2} = 7x^{1/4} - 2$

Sol. $5x^{1/2} - 7x^{1/4} + 2 = 0$ ……(i)

Let

$x^{1/4} = y$ ……(ii)

$x^{1/2} = y^2$.

Putting values in (ii)

$5y^2 - 7y + 2 = 0$ is quadratic equation

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$y - 1 = 0$, $5y - 2 = 0$

$y = 1$, $5y = 2$

$y = \frac{2}{5}$

From equation (ii)

$x^{1/4} = y$, $x^{1/4} = \frac{2}{5}$

$x^{1/4} = 1$, $x^{1/4} = \frac{2}{5}$

Taking power "4" on both sides

$(x^{1/4})^4 = (1)^4$, $(x^{1/4}) = \left(\frac{2}{5}\right)^4$

$x = 1$, $x = \frac{16}{625}$

Solution set $= \left\{1, \frac{16}{625}\right\}$

Q.4. $x^{2/3} + 54 = 15x^{1/3}$

Sol. $x^{2/3} - 15x^{1/3} + 54 = 0$ …….(i)

Let

$x^{1/3} = y$ …….(ii)

$(x^{1/3})^2 = y^2$

$x^{2/3} = y^2$ …….(iii)

Putting values in (i)

$y^2 - 15y + 54 = 0$ is quadratic equation

$y^2 - 6y - 9y + 54 = 0$

$y(y-6) - 9(y-6) = 0$

$(y-6)(y-9) = 0$

$y - 9 = 0$

$y = 9$

$y - 6 = 0$

$y = 6$

From (ii)

$x^{1/3} = y$

$x^{1/3} = 6$

From (ii)

$x^{1/3} = y$

$x^{1/3} = 9$

Taking power "3" on both side

$(x^{1/3})^3 = 9^3$ | $(x^{1/3})^3 = 6^3$

$x = 729$ | $x = 216$

Solution set $= \{729, 216\}$

Q.5. $3x^{-2} + 5 = 8x^{-1}$

Sol. $3x^{-2} - 8x^{-1} + 5 = 0$ …..(i)

Let $x^{-1} = y$ …..(ii)

$(x^{-1})^2 = y^2$

$x^{-2} = y^2$ …..(iii)

Putting values on (i)

$3y^2 - 8y + 5 = 0$ is quadratic equation

$3y^2 - 5y - 3y + 5 = 0$

$y(3y-5) - 1(3y-5) = 0$

$(y-1)(3y-5) = 0$

$y - 1 = 0$ | $3y - 5 = 0$

$y = 1$	$3y = 5$
	$y = \frac{5}{3}$

From (ii) $x^{-1} = y$	From (ii) $x^{-1} = y$
$x^{-1} = 1$	$x^{-1} = \frac{5}{3}$
$\frac{1}{x} = 1$	$\frac{1}{x} = \frac{5}{3}$
$1 = x.1$	$3 \times 1 = 5.x$
$1 = x$	$3 = 5x$
OR	$\frac{3}{5} = x$
$x = 1$	

Solution set $\left\{1, \frac{3}{5}\right\}$

Type (ii) : $\mathbf{ap(x) + \frac{b}{p(x)} = c}$

Q.6. $\mathbf{(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4}$

Sol. Let $2x^2 + 1 = y$ ……..(ii)

Put in equation ………(i)

$y + \frac{3}{y} = 4$

Multiplying both sides by "y"

$(y)(y) + y\left(\frac{3}{y}\right) = 4(y)$

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$ is quadratic equation

$y^2 - 3y - y + 3 = 0$

$y(y - 3) - 1(y - 3) = 0$

$(y - 1)(y - 3) = 0$

$y - 1 = 0$	$y - 3 = 0$
$y = 1$	$y = 3$

From equation (ii) $y = 2x^2 + 1$

$2x^2 + 1 = 1$	$2x^2 + 1 = 3$
$2x^2 = 1 - 1$	$2x^2 = 3 - 1$
$2x^2 = 0$	$2x^2 = 2$
$x^2 = \frac{0}{2}$	$x^2 = \frac{2}{2}$
$x^2 = 0$	$x^2 = 1$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{0}$	$\sqrt{x^2} = \sqrt{1}$
$x = 0$	$x = \pm 1$

Solution set $\{0, \pm 1\}$

Q.7. $\mathbf{\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4}$

Sol. $\frac{x}{x-3} + 4\left(\frac{1}{\left(\frac{x}{x-3}\right)}\right) = 4$ …….(i)

Let $\left(\frac{x}{x-3}\right) = y,$ ……(ii)

$y + 4\left(\frac{1}{y}\right) = 4$

Multiplying on both side by "y"

$y(y) + 4y\left(\frac{1}{y}\right) = (y)(4)$

$y^2 + y\left(\frac{4}{y}\right) = 4y$

$y^2 + 4 = 4y$

$y^2 - 4y + 4 = 0$ is quadratic equation

$y^2 - 2y - 2y + 4 = 0$

$y(y - 2) - 2(y - 2) = 0$

$(y - 2)(y - 2) = 0$

$y - 2 = 0 \Rightarrow y = 2$

Since both the answers are same. So, we solve any one. i.e; y = 2

From (ii) $y = \frac{x}{x-3}$

$\frac{x}{x-3} = 2$

$x = 2(x - 3)$

$x = 2x - 6$

$2x - x = 6$

$x = 6$

Solution set = $\{6\}$

Q.8. $\mathbf{\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}}$

Sol. $\frac{4x+1}{4x-1} + \left(\frac{1}{\left(\frac{4x+1}{4x-1}\right)}\right) = \frac{13}{6}$ ……. (i)

Let $\frac{4x+1}{4x-1} = y$ …… (ii)

$y + \left(\frac{1}{y}\right) = \frac{13}{6}$

Multiplying both sides by "y"

$y(y) + y\left(\frac{1}{y}\right) = \left(\frac{13}{6}\right)(y)$

$y^2 + 1 = \frac{13}{6} y$

$y^2 + 1 - \frac{13}{6} y = 0$

$y^2 - \frac{13}{6} y + 1 = 0$

Multiplying both sides by "6"

$6(y^2) - 6\left(\frac{13}{6} y\right) + 6(1) = 6(0)$

$6y^2 - 13y + 6 = 0$ is quadratic equation

$6y^2 - 9y - 4y + 6 = 0$

$3y(2y - 3) - 2(2y - 3) = 0$

$(3y - 2)(2y - 3) = 0$

$3y - 2 = 0$	$2y - 3 = 0$
$3y = 2$	$2y = 3$
$y = \frac{2}{3}$	$y = \frac{3}{2}$

From equation (ii)

$\frac{4x + 1}{4x - 1} = y$

$\frac{4x + 1}{4x - 1} = \frac{2}{3}$	$\frac{4x + 1}{4x - 1} = \frac{3}{2}$
$3(4x + 1) = 2(4x - 1)$	$2(4x + 1) = 3(4x - 1)$
$12x + 3 = 8x - 2$	$8x + 2 = 12x - 3$
$12x - 8x = -3 - 2$	$2 + 3 = 12x - 8x$
$4x = -5$	$5 = 4x$
$x = \frac{-5}{4}$	$x = \frac{5}{4}$

Solution set $= \{\pm\frac{5}{4}\}$

Q.9. $\mathbf{\frac{x - a}{x + a} - \frac{x + a}{x - a} = \frac{7}{12}}$

Sol. $\frac{x - a}{x + a} - \frac{x + a}{x - a} = \frac{7}{12}$

Let $\frac{x - a}{x + a} - \left(\frac{1}{\frac{x - a}{x + a}}\right) = \frac{7}{12}$……(i)

$\frac{x - a}{x + a} = y$ …..(ii)

$y - \frac{1}{y} = \frac{7}{12}$

Multiplying both sides by " y "

$y(y) - y\left(\frac{1}{y}\right) = \left(\frac{7}{12}\right)(y)$

$y^2 - 1 = \frac{7y}{12}$

$y^2 - \frac{7y}{12} - 1 = 0$

Multiplying by 12 on both sides

$12y^2 - 12\left(\frac{7y}{12}\right) - (12)(1) = 0(12)$

$12y^2 - 7y - 12 = 0$ is quadratic equation

$12y^2 - 16y + 9y - 12 = 0$

$4y(3y - 4) + 3(3y - 4) = 0$

$(4y + 3)(3y - 4) = 0$

$4y + 3 = 0$	$3y - 4 = 0$
$4y = -3$	$3y = 4$
$y = \frac{-3}{4}$	$y = \frac{4}{3}$

From (ii)

$\frac{x - a}{x + a} = \frac{-3}{4}$	$\frac{x - a}{x + a} = \frac{4}{3}$
$4(x - a) = -3(x + a)$	$3(x - a) = 4(x + a)$
$4x - 4a = -3x - 3a$	$3x - 3a = 4x + 4a$
$4x + 3x = -3a + 4a$	$-3a - 4a = 4x - 3x$
$7x = a$	$-7a = x$
$x = \frac{a}{7}$	

Solution set $= \{-7a, \frac{a}{7}\}$

Type (iii) $\mathbf{ax^4 + bx^3 + cx^2 + bx + a = 0}$ **OR**

$\mathbf{a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0}$

Q.10. $\mathbf{x^4 - 2x^3 - 2x^2 + 2x + 1 = 0}$

Sol. Dividing both side by "x^2"

$\frac{x^4}{x^2} - \frac{2x^3}{x^2} - \frac{2x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = \frac{0}{x^2}$

$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$

$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{1}{x}\right) - 2 = 0$ ….(i)

Let $x - \frac{1}{x} = y$ …(ii)

Taking square on both sides

$\left(x - \frac{1}{x}\right)^2 = y^2$

$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = y^2$

$x^2 + \frac{1}{x^2} - 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 + 2$ …..(iii)

Putting values in equation (i)

$(y^2 + 2) - 2(y) - 2 = 0$

$y^2 + 2 - 2y - 2 = 0$

$y^2 - 2y = 0$ is quadratic equation

$y(y - 2) = 0$

$y = 0 \quad , \quad y - 2 = 0$

$y = 0 \quad , \quad y = 2$

From equation (ii) $\quad y = x - \frac{1}{x}$

$x - \frac{1}{x} = 2 \quad , \quad x - \frac{1}{x} = 0$

Multiplying both side by "x"

$x \cdot x - \frac{1}{x} \cdot x = 2 \cdot x \; , \; x \cdot x - \frac{1}{x} \cdot x = 0 \cdot x$

$x^2 - 1 = 2x \quad , \quad x^2 - 1 = 0$

$x^2 - 2x - 1 = 0$

$\sqrt{x^2} = \sqrt{1} \quad \Rightarrow x = \pm 1$

Now compare with $\mathbf{ax^2 + bx + c = 0}$

$x^2 - 2x - 1 = 0$

$a = 1, \quad b = -2, \quad c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$= \frac{2 \pm \sqrt{4 + 4}}{2}$

$= \frac{2 \pm \sqrt{8}}{2}$

$= \frac{2 \pm \sqrt{4 \times 2}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$

$x = 1 \pm \sqrt{2}$

Solution set = $\{\pm 1, 1 \pm \sqrt{2}\}$

Q.11. $\mathbf{2x^4 + x^3 - 6x^2 + x + 2 = 0}$

Sol. Dividing both side by "x^2"

$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$

$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$

$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$

$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$ …….(i)

Let $\quad x + \frac{1}{x} = y$

Taking square on both sides

$\left(x + \frac{1}{x}\right)^2 = (y)^2$

$x^2 + \frac{1}{x^2} + 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 - 2$

Putting values in (i)

$2(y^2 - 2) + y - 6 = 0$

$2y^2 - 4 + y - 6 = 0$

$2y^2 + y - 10 = 0$ is quadratic equation

$2y^2 + 5y - 4y - 10 = 0$

$y(2y + 5) - 2(2y + 5) = 0$

$(y - 2)(2y + 5) = 0$

$y - 2 = 0 \quad , \quad 2y + 5 = 0$

$y = 2 \quad , \quad 2y = -5$

$\quad , \quad y = \frac{-5}{2}$

From equation (i)

$x + \frac{1}{x} = y \quad , \quad x + \frac{1}{x} = y$

$x + \frac{1}{x} = 2 \quad , \quad x + \frac{1}{x} = \frac{-5}{2}$

Multiplying on both sides by "x"

$x \cdot x + \frac{1}{x} \cdot x = 2$	$x \cdot x + \frac{1}{x} \cdot x = \frac{-5}{2} \cdot x$
$x^2 + 1 = 2.x$	$x^2 + 1 = \frac{-5}{2} x$
$x^2 + 1 = 2x$	$2(x^2 + 1) = -5x$
$x^2 - 2x + 1 = 0$	$2x^2 + 2 + 5x = 0$
Compare it with	$2x^2 + 5x + 2 = 0$
$ax^2 + bx + c = 0$	
$x^2 - 2x + 1 = 0$	$2x^2 + 4x + x + 2 = 0$
	$2x(x + 2) + 1(x + 2) = 0$
$a = 1$	$(2x + 1)(x + 2) = 0$
	$2x + 1 = 0 \; , \; x + 2 = 0$
$b = -2$	$2x = -1, \; x = -2$
$c = 1$	$x = \frac{-1}{2}$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-4}}{2}$$

$$= \frac{2 \pm \sqrt{0}}{2}$$

$$= \frac{2}{2} = 1$$

Solution set $= \left\{1, -2, -\frac{1}{2}\right\}$

Type (iv) Exponential Equations:

In exponential equation, variable occurs in exponential form. OR is exponent of constant value e.g;

$a.k^{2x} + b.k^x + c = 0$

Note: $x^o = 1,\ k^x = k^n$

$\Rightarrow\ x = n,\ (a^m)^n = a^{mn},\ a^m.a^n = a^{m+n}$

Q.12. $4.2^{2x+1} - 9.2^x + 1 = 0$

Sol. $4.2^{2x}.\,2 - 9.2^x + 1 = 0$

$8.(2^{2x}) - 9.2^x + 1 = 0$ …….(i)

Let $2^x = y$ …….(ii)

$(2^x)^2 = (y)^2$

$2^{2x} = y^2$ …….(iii)

Putting values in (i)1

$8y^2 - 9y + 1 = 0$ is quadratic equation

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(8y-1)(y-1) = 0$

$8y - 1 = 0$	$y - 1 = 0$
$8y = 1$	$y = 1$
$y = \frac{1}{8}$	

Putting values in (ii)

$2^x = y$	$2^x = y$
$2^x = \frac{1}{8}$	$2^x = 1$
$2^x = \frac{1}{2^3}$	$2^x = 2^0$
	$\because\ 2^o = 1$
$2^x = 2^{-3}$	$x = 0$
$x = -3$	

Solution set $= \{-3, 0\}$

Q.13. $3^{2x+2} = 12.3^x - 3$

Sol. $3^{2x}.3^2 - 12.3^x + 3 = 0$

$9.3^{2x} - 12.3^x + 3 = 0$

$9.(3^x)^2 - 12.3^x + 3 = 0$ …….. (i)

Let $3^x = y$ …….. (ii)

$3^{2x} = y^2$ ……..(iii)

Putting values in (i)

$9y^2 - 12y + 3 = 0$ is quadratic equation

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(9y-3)(y-1) = 0$

$9y - 3 = 0$	$y - 1 = 0$
$9y = 3$	$y = 1$
$y = \frac{3}{9}$	
$y = \frac{1}{3}$	

Putting values in (ii)

$3^x = y$	$3^x = y$
$3^x = \frac{1}{3}$	$3^x = 1$
	$3^x = 3^0$
$3^x = 3^{-1}$	$\because\ 3^o = 1$
$x = -1$	$x = 0$

Solution set $= \{-1, 0\}$

Q.14. $2^x + 64.2^{-x} - 20 = 0$

Sol. $2^x + \frac{64}{2^x} - 20 = 0$ …………(i)

Let $2^x = y$

Put in …….(i)

$y + \frac{64}{y} - 20 = 0$

Multiplying both sides by "y"

$y(y) + y\left(\frac{64}{y}\right) - (y)(20) = y(0)$

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$ is quadratic equation

$y^2 - 16y - 4y + 64 = 0$

$y(y-16) - 4(y-16) = 0$

$(y-16)(y-4) = 0$

$y - 4 = 0$	$y - 16 = 0$
$y = 4$	$y = 16$

Put value in (i)

$2^x = y$	$2^x = y$
$2^x = 4$	$2^x = 16$
$2^x = 2^2$	$2^x = 2^4$
$x = 2$	$x = 4$

Solution set $= \{2, 4\}$

Type v: (x + a) (x + b) (x + c) (x + d) = k

Where a + b = c + d

Q.15. $(x + 1)(x + 3)(x - 5)(x - 7) = 192$

Sol. $(x + 1)(x - 5)(x + 3)(x - 7) - 192 = 0$

$1 - 5 = -4 \quad (\because 1 - 5 = 3 - 7)$

$3 - 7 = -4$

$[(x + 1)(x - 5)][(x + 3)(x - 7)] - 192 = 0$

$(x^2 - 5x + x - 5)(x^2 - 7x + 3x - 21) - 192 = 0$

$(x^2 - 4x - 5)(x^2 - 4x - 21) - 192 = 0$

Let $x^2 - 4x = y$

$(y - 5)(y - 21) - 192 = 0$

$y^2 - 5y - 21y + 105 - 192 = 0$

$y^2 - 26y - 87 = 0$ is quadratic equation

$y^2 - 29y + 3y - 87 = 0$

$y(y - 29) + 3(y - 29) = 0$

$(y + 3)(y - 29) = 0$

$y + 3 = 0 \quad | \quad y - 29 = 0$

$y = -3 \quad | \quad y = 29$

From (ii)

$y = x^2 - 4x$

$x^2 - 4x = -3$

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x - 3) - 1(x - 3) = 0$

$(x - 1)(x - 3) = 0$

$x - 1 = 0 \quad | \quad x - 3 = 0$

$x = 1 \quad | \quad x = 3$

$x^2 - 4x = 29$

$x^2 - 4x - 29 = 0$

Compare with $ax^2 + bx + c = 0$

$a = 1, b = -4, c = -29$

Quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 4(-29)}}{2(1)} = \frac{4 \pm \sqrt{16 + 116}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 116}}{2(1)} = \frac{4 \pm \sqrt{132}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 33}}{2} = \frac{4 \pm 2\sqrt{33}}{2}$$

$$= \frac{2(2 \pm \sqrt{33})}{2} = 2 \pm \sqrt{33}$$

Solution set $\{1, 3, 2 \pm \sqrt{33}\}$

Q.16. $(x - 1)(x - 2)(x - 8)(x + 5) + 360 = 0$

$-1 - 2 = -8 + 5 = -3$

Sol. $[(x - 1)(x - 2)][(x - 8)(x + 5)] + 360 = 0$

$(x^2 - 2x - x + 2)(x^2 + 5x - 8x - 40) + 360 = 0$

$(x^2 - 3x + 2)(x^2 - 3x - 40) + 360 = 0$

Let $x^2 - 3x = y$ ……... (ii)

$(y + 2)(y - 40) + 360 = 0$

$y^2 - 40y + 2y - 80 + 360 = 0$

$y^2 - 38y + 280 = 0$ is quadratic equation

$y^2 - 28y - 10y + 280 = 0$

$y(y - 28) - 10(y - 28) = 0$

$(y - 10)(y - 28) = 0$

$y - 10 = 0 \quad | \quad y - 28 = 0$

$y = 10 \quad | \quad y = 28$

From (ii)

$x^2 - 3x = y$	$x^2 - 3x = y$
$x^2 - 3x = 10$	$x^2 - 3x = 28$
$x^2 - 3x - 10 = 0$	$x^2 - 3x - 28 = 0$
$x^2 - 5x + 2x - 10 = 0$	$x^2 - 7x + 4x - 28 = 0$
$x(x - 5) + 2(x - 5) = 0$	$x(x - 7) + 4(x - 7) = 0$
$(x + 2)(x - 5) = 0$	$(x + 4)(x - 7) = 0$
$x + 2 = 0 \mid x - 5 = 0$	$x + 4 = 0 \mid x - 7 = 0$
$x = -2 \mid x = 5$	$x = -4 \mid x = 7$

Solution set = $\{-4, 5, -2, 7\}$

Radical Equation:

An equation involving expression under the radical sign is called a **Radical Equation**

e.g; $\sqrt{x + 3} = x + 1$ and $\sqrt{x - 1} = \sqrt{x - 2} + 1$

Extraneous root:

"A root of an equation, which does not satisfy the given equation is **called extraneous root."**

Note: For solving Radical Equations, Squaring both sides in proper way because equations free from radical signs.

EXERCISE 1.4

(i) Equation of Type: $\sqrt{ax + b} = cx + d$

Q.1 Solve the following equations.

$2x + 5 = \sqrt{7x + 16}$

Sol. Squaring on both sides

$(2x + 5)^2 = \left(\sqrt{7x + 16}\right)^2$

$(2x)^2 + (5)^2 + 2(2x)(5) = 7x + 16$

$4x^2 + 25 + 20x = 7x + 16$

$4x^2 + 20x - 7x + 25 - 16 = 0$

$4x^2 + 13x + 9 = 0$

$4x^2 + 4x + 9x + 9 = 0$

$4x(x + 1) + 9(x + 1) = 0$

$(4x + 9)(x + 1) = 0$

$4x + 9 = 0 \qquad x + 1 = 0$

$x = \frac{-9}{4} \qquad x = -1$

Check

Put $x = -1$	put $x = \frac{-9}{4}$
$2x + 5 = \sqrt{7x + 16}$	$2x + 5 = \sqrt{7x + 16}$
$2(-1) + 5 = \sqrt{7(-1) + 16}$	$2\left(\frac{-9}{4}\right) + 5 = \sqrt{7\left(\frac{-9}{4}\right) + 16}$
$-2 + 5 = \sqrt{-7 + 16}$	$-\frac{9}{2} + 5 = \sqrt{\frac{-63}{4} + 16}$
$3 = \sqrt{9}$	$\frac{-9 + 10}{2} = \sqrt{\frac{-63 + 64}{4}}$
$3 = \sqrt{3^2}$	$\frac{1}{2} = \sqrt{\frac{1}{4}}$
$3 = 3$	$\frac{1}{2} = \frac{1}{2}$

Solution set = $\{-1, \frac{-9}{4}\}$

Q.2. $\sqrt{x + 3} = 3x - 1$

Sol. Squaring on both sides

$$\left(\sqrt{x + 3}\right)^2 = (3x - 1)^2$$
$$x + 3 = (3x)^2 + (1)^2 - 2(3x)(1)$$
$$x + 3 = 9x^2 + 1 - 6x$$
$$x + 3 = 9x^2 - 6x + 1$$
$$0 = 9x^2 - 6x - x - 3 + 1$$
$$0 = 9x^2 - 7x - 2$$
$$9x^2 - 7x - 2 = 0$$
$$9x^2 - 9x + 2x - 2 = 0$$
$$9x(x - 1) + 2(x - 1) = 0$$
$$(9x + 2)(x - 1) = 0$$
$$9x + 2 = 0 \qquad x - 1 = 0$$
$$9x = -2 \qquad x = 1$$
$$x = \frac{-2}{9}$$

Check

$\sqrt{x + 3} = 3x - 1$	$\sqrt{x + 3} = 3x - 1$
Put $x = \frac{-2}{9}$	$x = 1$
$\sqrt{\frac{-2}{9} + 3} = 3\left(\frac{-2}{9}\right) - 1$	$\sqrt{1 + 3} = 3(1) - 1$
$\sqrt{\frac{-2 + 27}{9}} = \frac{-2}{3} - 1$	$\sqrt{4} = 3 - 1$
$\sqrt{\frac{25}{9}} = \frac{-2-3}{3}$	$2 = 2$
$\frac{5}{3} \neq \frac{-5}{3}$	

Solution set = $\{+1\}\left(\frac{-2}{9}\text{ extraneous}\right)$

Q.3. $4x = \sqrt{13x + 14} - 3$

Sol. $4x + 3 = \sqrt{13x + 14}$

Squaring on both sides

$$(4x + 3)^2 = \left(\sqrt{13x + 14}\right)^2$$
$$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$$
$$16x^2 + 9 + 24x = 13x + 14$$
$$16x^2 + 24x - 13x + 9 - 14 = 0$$
$$16x^2 + 11x - 5 = 0$$
$$16x^2 + 16x - 5x - 5 = 0$$
$$16x(x + 1) - 5(x + 1) = 0$$
$$(16x - 5)(x + 1) = 0$$
$$16x - 5 = 0 \qquad x + 1 = 0$$
$$x = \frac{5}{16} \qquad x = -1$$

Check

$4x = \sqrt{13x + 14} - 3$	$4x = \sqrt{13x + 14} - 3$
Put $x = \frac{5}{16}$	Put $x = -1$
$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3$	$4(-1) = \sqrt{13(-1) + 14} - 3$
$\frac{5}{4} = \sqrt{\frac{65}{16} + 14} - 3$	$-4 = \sqrt{-13 + 14} - 3$
$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$	$-4 = \sqrt{1} - 3$
$\frac{5}{4} = \frac{17}{4} - 3$	$-4 = 1 - 3$
$\frac{5}{4} = \frac{17 - 12}{4}$	$-4 \neq -2$
$\frac{5}{4} = \frac{5}{4}$	

Solution set = $\left\{\frac{5}{16}\right\}$ (extraneous -1)

Q.4. $\sqrt{3x + 100} - x = 4$

Sol. $\sqrt{3x + 100} = 4 + x$

Taking square on both sides

$$\left(\sqrt{3x + 100}\right)^2 = (4 + x)^2$$
$$3x + 100 = (4)^2 + (x)^2 + 2(4)(x)$$
$$3x + 100 = 16 + (x)^2 + 8x$$
$$0 = 8x - 3x - 100 + 16 + x^2$$
$$0 = 5x - 84 + x^2$$

$0 = x^2 + 5x - 84$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, \quad b = 5, \quad c = -84$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(-84)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{25 + 336}}{2}$

$x = \dfrac{-5 \pm \sqrt{361}}{2}$

$x = \dfrac{-5 \pm 19}{2}$

$x = \dfrac{-5 + 19}{2}$	$x = \dfrac{-5 - 19}{2}$
$x = \dfrac{14}{2}$	$x = \dfrac{-24}{2}$
$x = 7$	$x = -12$

Check

$\sqrt{3x + 100} - x = 4$	$\sqrt{3x + 100} - x = 4$
Put x = 7	Put x = - 1 2
$\sqrt{3(7) + 100} - 7 = 4$	$\sqrt{3(-12) + 100} - x = 4$
$\sqrt{21 + 100} - 7 = 4$	$\sqrt{-36 + 100} - (-12) = 4$
$\sqrt{121} - 7 = 4$	$\sqrt{64} + 12 = 4$
$11 - 7 = 4$	$8 + 12 = 4$
$4 = 4$	$20 \neq 4$

Solution set = {7} (–12 extraneous)

Type II: $\sqrt{x + a} + \sqrt{x + b} = \sqrt{x + c}$

Q.5. $\sqrt{x + 5} + \sqrt{x + 21} = \sqrt{x + 60}$

Sol. Squaring on both sides

$\left(\sqrt{x + 5} + \sqrt{x + 21}\right)^2 = \left(\sqrt{x + 60}\right)^2$

$\left(\sqrt{x + 5}\right)^2 + \left(\sqrt{x + 21}\right)^2 + 2\left(\sqrt{x + 5}\right)\left(\sqrt{x + 21}\right) = \left(\sqrt{x + 60}\right)^2$

$x + 5 + x + 21 + 2\sqrt{x^2 + 21x + 5x + 105} = x + 60$

$2x + 26 + 2\sqrt{x^2 + 26x + 105} = x + 60$

$2\sqrt{x^2 + 26x + 105} = x + 60 - 2x - 26$

$2\sqrt{x^2 + 26x + 105} = 34 - x$

Taking square on both sides

$\left(2\sqrt{x^2 + 26x + 105}\right)^2 = (34 - x)^2$

$4(x^2 + 26x + 105) = (34)^2 + (x)^2 - 2(34)(x)$

$4x^2 + 104x + 420 = 1156 + x^2 - 68x$

$4x^2 - x^2 + 104x + 68x + 420 - 1156 = 0$

$3x^2 + 172x - 736 = 0$

$3x^2 + 184x - 12x - 736 = 0$

$x(3x + 184) - 4(3x + 184) = 0$

$(x - 4)(3x + 184) = 0$

$x - 4 = 0 \qquad 3x + 184 = 0$

$x = 4 \qquad x = \dfrac{-184}{3}$

Check

$\sqrt{x + 5} + \sqrt{x + 21} = \sqrt{x + 60}$

Put $x = 4$

$\sqrt{4 + 5} + \sqrt{4 + 21} = \sqrt{4 + 60}$

$\sqrt{9} + \sqrt{25} = \sqrt{64}$

$3 + 5 = 8$

$8 = 8$

$\sqrt{x + 5} + \sqrt{x + 21} = \sqrt{x + 60}$

Put $x = \dfrac{-184}{3}$

$\sqrt{\dfrac{-184}{3} + 5} + \sqrt{\dfrac{-184}{3} + 21} = \sqrt{\dfrac{-184}{3} + 60}$

$\sqrt{\dfrac{-184 + 15}{3}} + \sqrt{\dfrac{-184 + 63}{3}} = \sqrt{\dfrac{-184 + 180}{3}}$

$\sqrt{\dfrac{-169}{3}} + \sqrt{\dfrac{-121}{3}} = \sqrt{\dfrac{-4}{3}}$

$\sqrt{\dfrac{169(-1)}{3}} + \sqrt{\dfrac{121(-1)}{3}} = \sqrt{\dfrac{4(-1)}{3}}$

$\sqrt{\dfrac{-169}{3}} + \dfrac{11i}{\sqrt{3}} = \dfrac{2i}{\sqrt{3}}$

$\because \quad (i)^2 = (-1)$

$\dfrac{13i}{\sqrt{3}} + \dfrac{11i}{\sqrt{3}} \neq \dfrac{2i}{\sqrt{3}}$

$\dfrac{24i}{\sqrt{3}} \neq \dfrac{2i}{\sqrt{3}}$

Solution set = {4} $\left(\dfrac{-184}{3}\text{ extraneous}\right)$

Q.6. $\sqrt{x + 1} + \sqrt{x - 2} = \sqrt{x + 6}$

Sol. Taking square on both sides

$\left(\sqrt{x + 1} + \sqrt{x - 2}\right)^2 = \left(\sqrt{x + 6}\right)^2$

$(\sqrt{x + 1})^2 + (\sqrt{x - 2})^2 + 2(\sqrt{x + 1}\ \sqrt{x - 2}) = x + 6$

$x + 1 + x - 2 + 2\left(\sqrt{(x + 1)(x - 2)}\right) = x + 6$

$2x - 1 + 2\left(\sqrt{(x + 1)(x - 2)}\right) = x + 6$

$2\sqrt{(x + 1)(x - 2)} = x + 6 - 2x + 1$

$2\sqrt{x^2 - x - 2} = 7 - x$

Squaring on both sides

$\left(2\sqrt{x^2 - x - 2}\right)^2 = (7 - x)^2$

$4(x^2 - x - 2) = (7)^2 + (x)^2 - 2(7)(x)$

$4x^2 - 4x - 8 = 49 + x^2 - 14x$

$4x^2 - x^2 - 4x + 14x - 8 - 49 = 0$

$3x^2 + 10x - 57 = 0$

$3x^2 + 19x - 9x - 57 = 0$

$x(3x + 19) - 3(3x + 19) = (x - 3)(3x + 19) = 0$

$x - 3 = 0 \qquad 3x + 19 = 0$

$x = 3 \qquad x = \frac{-19}{3}$

Check

$\sqrt{x + 1} + \sqrt{x - 2} = \sqrt{x + 6}$

Put $x = 3$

$\sqrt{3 + 1} + \sqrt{3 - 2} = \sqrt{3 + 6}$

$\sqrt{4} + \sqrt{1} = \sqrt{9}$

$2 + 1 = 3$

$3 = 3$

$\sqrt{x + 1} + \sqrt{x - 2} = \sqrt{x + 6}$

Put $x = x = \frac{-19}{3}$

$\sqrt{\frac{-19}{3} + 1} + \sqrt{\frac{-19}{3} - 2} = \sqrt{\frac{-19}{3} + 6}$

$\sqrt{\frac{-19 + 3}{3}} + \sqrt{\frac{-19 - 6}{3}} = \sqrt{\frac{-19 + 18}{3}}$

$\sqrt{\frac{-16}{3}} + \sqrt{\frac{-25}{3}} = \sqrt{\frac{-1}{3}}$

$\sqrt{\frac{16(-1)}{3}} + \sqrt{\frac{25(-1)}{3}} = \sqrt{\frac{-1}{3}}$

$\sqrt{\frac{16(i)^2}{3}} + \sqrt{\frac{25(i)^2}{3}} = \sqrt{\frac{(i)^2}{3}}$

$\because \quad i^2 = -1$

$\frac{4i}{\sqrt{3}} + \frac{5i}{\sqrt{3}} \neq \frac{i}{\sqrt{3}}$

$\frac{9i}{\sqrt{3}} \neq \frac{i}{\sqrt{3}}$

Solution set $\{3\}$ $\left(\text{extraneous } \frac{-19}{3}\right)$

Q.7. $\sqrt{\mathbf{11 - x}} - \sqrt{\mathbf{6 - x}} = \sqrt{\mathbf{27 - x}}$

Sol. Squaring both sides

$\left(\sqrt{11 - x} - \sqrt{6 - x}\right)^2 = \left(\sqrt{27 - x}\right)^2$

$(\sqrt{11 - x})^2 + (\sqrt{6 - x})^2 - 2\sqrt{11 - x}\sqrt{6 - x} = 27 - x$

$11 - x + 6 - x - 2\sqrt{66 - 11x - 6x + x^2} = 27 - x$

$17 - 2x + 2\sqrt{66 - 11x - 6x + x^2} = 27 - x$

$2\sqrt{66 - 17x + x^2} = 27 - x - 17 + 2x$

$2\sqrt{66 - 17x + x^2} = 10 + x$

Squaring both sides

$(2\sqrt{66 - 17x + x^2})^2 = (10 + x)^2$

$4(66 - 17x + x^2) = (10)^2 + (x)^2 + 2(10)(x)$

$264 - 68x + 4x^2 = 100 + x^2 + 20x$

$264 - 68x + 4x^2 - 100 - x^2 - 20x = 0$

$3x^2 - 88x + 164 = 0$

$3x^2 - 82x - 6x + 164 = 0$

$x(3x - 82) - 2(3x - 82) = 0$

$(x - 2)(3x - 82) = 0$

$x - 2 = 0 \qquad 3x - 82 = 0$

$x = 2 \qquad x = \frac{82}{3}$

Check

$\sqrt{11 - x} + \sqrt{6 - x} = \sqrt{27 - x}$

Put $x = 2$

$\sqrt{11 - 2} + \sqrt{6 - 2} = \sqrt{27 - 2}$

$\sqrt{9} + \sqrt{4} = \sqrt{25}$

$3 + 2 = 5$

$5 = 5$

$\sqrt{11 - x} + \sqrt{6 - x} = \sqrt{27 - x}$

Put $x = \frac{82}{3}$

$\sqrt{11 - \frac{82}{3}} + \sqrt{6 - \frac{82}{3}} + \sqrt{27 - \frac{82}{3}}$

$\sqrt{\frac{33 - 82}{3}} + \sqrt{\frac{18 - 82}{3}} = \sqrt{\frac{81 - 82}{3}}$

$\sqrt{\frac{-49}{3}} + \sqrt{\frac{-64}{3}} = \sqrt{\frac{-1}{3}}$

$\sqrt{\frac{49(-1)}{3}} + \sqrt{\frac{64(-1)}{3}} = \sqrt{\frac{-1}{3}}$

$\sqrt{\frac{49(i)^2}{3}} + \sqrt{\frac{64(i)^2}{3}} = \sqrt{\frac{(i)^2}{3}}$

$\because \quad i^2 = -1$

$\frac{7i}{\sqrt{3}} + \frac{8i}{\sqrt{3}} \neq \frac{i}{\sqrt{3}}$

$\frac{15i}{\sqrt{3}} \neq \frac{i}{\sqrt{3}}$

Solution set = {2} $\left(\text{extraneous } \frac{82}{3}\right)$

Q.8. $\sqrt{4a + x} - \sqrt{a - x} = \sqrt{a}$

Sol. Taking square on both sides

$(\sqrt{4a+x} - \sqrt{a-x})^2 = (\sqrt{a})^2$

$(\sqrt{4a+x})^2 + (\sqrt{a-x})^2 - 2(\sqrt{4a+x})(\sqrt{a-x}) = a$

$4a + x + a - x - 2\sqrt{(4a+x)(a-x)} = a$

$5a - 2\sqrt{4a^2 - 4ax - x^2 + ax} = a$

$-2\sqrt{4a^2 - 4ax - x^2 + ax} = a - 5a$

$-2\sqrt{4a^2 - 4ax - x^2 + ax} = -4a$

Squaring both sides

$(-2\sqrt{4a^2 - 3ax - x^2})^2 = (-4a)^2$

$4(4a^2 - 3ax - x^2) = 16a^2$

$16a^2 - 12ax - 4x^2 = 16a^2$

$16a^2 - 12ax - 4x^2 - 16a^2 = 0$

$-12ax - 4x^2 = 0$

$4x(-3a - x) = 0$

$4x = 0 \qquad -3a - x = 0$

$x = \frac{0}{4} \qquad -3a = x$

$x = 0 \qquad x = -3a$

Check

$\sqrt{4a + x} - \sqrt{a - x} = \sqrt{a}$

Put x = 0

$\sqrt{4a + 0} - \sqrt{a - 0} = \sqrt{a}$

$\sqrt{4a} - \sqrt{a} = \sqrt{a}$

$2\sqrt{a} - \sqrt{a} = \sqrt{a}$

$(2 - 1)\sqrt{a} = \sqrt{a}$

$\sqrt{a} = \sqrt{a}$

$\sqrt{4a + x} - \sqrt{a - x} = \sqrt{a}$

Put x = – 3a

$\sqrt{4a - 3a} - \sqrt{a + 3a} = \sqrt{a}$

$\sqrt{a} - \sqrt{4a} = \sqrt{a}$

$\sqrt{a} - 2\sqrt{a} = \sqrt{a}$

$\sqrt{a}(1 - 2) = \sqrt{a}$

$-\sqrt{a} \neq \sqrt{a}$

Solution set = {0} (extraneous –3a}

Type III: $\sqrt{x^2 + px + m} + \sqrt{x^2 + px + n} = q$

let $y = x^2 + px$

Q.9. $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1$

Sol. Let $x^2 + x = y$ ……..(i)

Putting values

$\sqrt{y+1} - \sqrt{y-1} = 1$

Squaring on both sides

$(\sqrt{y+1} - \sqrt{y-1})^2 = (1)^2$

$(\sqrt{y+1})^2 + (\sqrt{y-1})^2 - 2(\sqrt{y+1})(\sqrt{y-1}) = 1$

$\because \quad (a - b)^2 = a^2 + b^2 - 2ab$

$y + 1 + y - 1 - 2\sqrt{(y+1)(y-1)}$

$2y - 2\sqrt{y^2 - 1} = 1$

$\because \quad (a - b)(a + b) = a^2 - b^2$

$(-2\sqrt{y^2 - 1}) = -2y$

Squaring both sides

$(-2\sqrt{y^2 - 1})^2 = (1 - 2y)^2$

$4(y^2 - 1) = (1)^2 + (2y)^2 - 2(1)(2y)$

$4y^2 - 4 = 1 + 4y^2 - 4y$

$4y^2 - 4 - 1 - 4y^2 + 4y = 0$

$-5 + 4y = 0$

$4y - 5 = 0$

$4y = 5$

$y = \frac{5}{4}$

From equation (1) $x^2 + x = y$

$x^2 + x = \frac{5}{4}$

$4(x^2 + x) = 5$

$4x^2 + 4x - 5 = 0$

By comparing with

$ax^2 + bx + c = 0$

a = 4, b = 4, c = –5

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$

$x = \frac{-4 \pm \sqrt{16 + 80}}{8} = \frac{-4 \pm \sqrt{96}}{8}$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$\because \quad \sqrt{16 \times 6} = \sqrt{96}, \sqrt{16} \times \sqrt{6} = 4\sqrt{6}$

$$x = \frac{4(-1 \pm \sqrt{6})}{8} = \frac{(-1 \pm \sqrt{6})}{2}$$

$$x = \frac{-1+\sqrt{6}}{2}, \quad x = \frac{-1-\sqrt{6}}{2}$$

Check: putting $x = \frac{-1+\sqrt{6}}{2}$ in

$$\sqrt{x^2+x+1} - \sqrt{x^2+x-1}$$

$$\sqrt{\left[\frac{-1+\sqrt{6}}{2}\right]^2 + \frac{-1+\sqrt{6}}{2} + 1} - \sqrt{\left[\frac{-1+\sqrt{6}}{2}\right]^2 + \frac{-1+\sqrt{6}}{2} - 1} = 1$$

$$\sqrt{\frac{(-1)^2 + (\sqrt{6})^2 - 2(1)(\sqrt{6})}{4} + \frac{-1+\sqrt{6}}{2} + 1} - \sqrt{\frac{(-1)^2 + (\sqrt{6})^2 - 2(1)(\sqrt{6})}{4} + \frac{-1+\sqrt{6}}{2} - 1} = 1$$

$$\sqrt{\frac{1+6-2\sqrt{6}}{4} + \frac{-1+\sqrt{6}}{2} + 1} - \sqrt{\frac{1+6-2\sqrt{6}}{4} + \frac{-1+\sqrt{6}}{2} - 1} = 1$$

$$\sqrt{\frac{(7-2\sqrt{6})+2(-1+\sqrt{6})+4}{4}} - \sqrt{\frac{(7-2\sqrt{6})+2(-1+\sqrt{6})-4}{4}} = 1$$

$$\sqrt{\frac{7-2\sqrt{6}-2+2\sqrt{6}+4}{4}} - \sqrt{\frac{7-2\sqrt{6}-2+2\sqrt{6}-4}{4}} = 1$$

$$\sqrt{\frac{9}{4}} - \sqrt{\frac{1}{4}} = 1$$

$$\Rightarrow \frac{3}{2} - \frac{1}{2} = 1$$

$$\frac{2}{2} = 1$$

$$1 = 1$$

So, L.H.S = R.H.S

Check: $x = \frac{-1-\sqrt{6}}{2}$

$$\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$$

$$\sqrt{\left[\frac{-1-\sqrt{6}}{2}\right]^2 + \left[\frac{-1-\sqrt{6}}{2}\right] + 1} - \sqrt{\left[\frac{-1-\sqrt{6}}{2}\right]^2 + \left[\frac{-1-\sqrt{6}}{2}\right] - 1} = 1$$

$$\sqrt{\frac{(-1)^2 + (-\sqrt{6})^2 + 2(-1)(-\sqrt{6})}{4} + \frac{(-1-\sqrt{6})}{2} + 1} - \sqrt{\frac{(-1)^2 + (-\sqrt{6})^2 + 2(-1)(-\sqrt{6})}{4} + \frac{(-1-\sqrt{6})}{2} - 1} = 1$$

$$\sqrt{\frac{1+6+2\sqrt{6}}{4} + \frac{(-1-\sqrt{6})}{2} + 1} - \sqrt{\frac{1+6+2\sqrt{6}}{4} + \frac{(-1-\sqrt{6})}{2} - 1} = 1$$

$$\sqrt{\frac{(7+2\sqrt{6})+2(-1-1\sqrt{6})+4}{4}} - \sqrt{\frac{(7+2\sqrt{6})+2(-1-1\sqrt{6})-4}{4}} = 1$$

$$\sqrt{\frac{7+2\sqrt{6}-2-2\sqrt{6}+4}{4}} - \sqrt{\frac{7+2\sqrt{6}-2-2\sqrt{6}-4}{4}} = 1$$

$$\sqrt{\frac{9}{4}} - \sqrt{\frac{1}{4}} = 1$$

$$\frac{3}{2} - \frac{1}{2} = 1 \Rightarrow \frac{3-1}{2} = 1$$

$$\frac{2}{2} = 1$$

$$\Rightarrow 1 = 1$$

Solution set = $\left\{\frac{-1 \pm \sqrt{6}}{2}\right\}$

So, L.H.R = R.H.S

Q.10. $\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$

Sol. Let

$\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$ ….(i)

$x^2 + 3x = y$

Put in (i)

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = (3)^2$$

$$\left(\sqrt{y+8}\right)^2 + \left(\sqrt{y+2}\right)^2 + 2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right) = 9$$

$\because \quad (a + b)^2 = a^2 + b^2 + 2ab$

$y + 8 + y + 2 + 2\sqrt{(y + 8)(y + 2)} = 9$

$(2\sqrt{y^2 + 2y + 8y + 16}) = 9 - y - 8 - y - 2$

Squaring both sides

$(2\sqrt{y^2 + 10y + 16})^2 = (-2y - 1)^2$

$4(y^2 + 10y + 16) = (-2y)^2 + (-1)^2 + 2(-2y)(-1)$

$4y^2 + 40y + 64 = 4y^2 + 1 + 4y$

$4y^2 - 4y^2 + 40y - 4y + 64 - 1 = 0$

$36y + 63 = 0$

$y = \frac{-63}{36} \Rightarrow y = \frac{-7}{4}$

$x^2 + 3x = y \Rightarrow x^2 + 3x = -\frac{7}{4}$

$4(x^2 + 3x) = -7$

$4x^2 + 12x = -7$

$4x^2 + 12x + 7 = 0$

Compare with $ax^2 + bx + c = 0$

$a = 4, \quad b = 12, \quad c = 7$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-12 \pm \sqrt{(12)^2 - 4(4)(7)}}{2(4)}$

$= \frac{-12 \pm \sqrt{144 - 112}}{8}$

$= \frac{-12 \pm \sqrt{32}}{8} = \frac{-12 \pm \sqrt{16 \times 2}}{8}$

$= \frac{-12 \pm 4\sqrt{2}}{8} = \frac{4(-3 \pm \sqrt{2})}{8}$

$= \frac{(-3 \pm \sqrt{2})}{2}$

Checking:

$\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$

putting $x = \frac{\left(-3 + \sqrt{2}\right)}{2}$

$$\sqrt{\left(\frac{-3+\sqrt{2}}{2}\right)^2 + 3\left(\frac{-3+\sqrt{2}}{2}\right) + 8}$$

$$+\sqrt{\left(\frac{-3+\sqrt{2}}{2}\right)^2 + 3\left(\frac{-3+\sqrt{2}}{2}\right) + 2} = 3$$

$$\sqrt{\frac{(-3)^2 + 2(-3)(\sqrt{2}) + (\sqrt{2})^2}{(2)^2} + \frac{-9+3\sqrt{2}}{2} + 8}$$

$$+\sqrt{\frac{(-3)^2 + 2(-3)(\sqrt{2}) + (\sqrt{2})^2}{(2)^2} + \frac{-9+3\sqrt{2}}{2} + 2} = 3$$

$$\sqrt{\frac{9 - 6\sqrt{2} + 2}{4} + \frac{-9+3\sqrt{2}}{2} + 8}$$

$$+\sqrt{\frac{9 - 6\sqrt{2} + 2}{4} + \frac{-9+3\sqrt{2}}{2} + 2} = 3$$

$$\sqrt{\frac{9 - 6\sqrt{2} + 2 + 2\left(-9 + 3\sqrt{2}\right) + 32}{4}}$$

$$+\sqrt{\frac{9 - 6\sqrt{2} + 2 + 2\left(-9 + 3\sqrt{2}\right) + 8}{4}} = 3$$

$$\sqrt{\frac{9 - 6\sqrt{2} + 2 - 18 + 6\sqrt{2} + 32}{4}}$$

$$+\sqrt{\frac{9 - 6\sqrt{2} + 2 - 18 + 6\sqrt{2} + 8}{4}} = 3$$

$$\sqrt{\frac{9 + 2 - 18 + 32}{4}} + \sqrt{\frac{9 + 2 - 18 + 8}{4}} = 3$$

$$\sqrt{\frac{25}{4}} + \sqrt{\frac{1}{4}} = 3 \Rightarrow \frac{5}{2} + \frac{1}{2} = 3$$

$$\frac{5+1}{2} = 3 \Rightarrow \frac{6}{2} = 3$$

$3 = 3$

Q.11. $\sqrt{\mathbf{x^2 + 3x + 9}} + \sqrt{\mathbf{x^2 + 3x + 4}} = \mathbf{5}$

Sol. Let

Put $x^2 + 3x = y$ ……(i)

$\sqrt{y + 9} + \sqrt{y + 4} = 5$

Squaring on both sides

$\left(\sqrt{y + 9} + \sqrt{y + 4}\right)^2 = (5)^2$

$\left(\sqrt{y + 9}\right)^2 + \left(\sqrt{y + 4}\right)^2$

$+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right) = 25$

$\left(\sqrt{y+9}\right)^2 + \left(\sqrt{y+4}\right)^2$

$+ 2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right) = 25$

$\because (a+b)^2 = a^2+b^2+2ab$

$y + 9 + y + 4 + 2\left(\sqrt{(y+9)(y+4)}\right) = 25$

$2y + 13 + 2\sqrt{y^2+13y+36} = 25$

$2\sqrt{y^2+13y+36} = 25 - 2y - 13$

$2\sqrt{y^2+13y+36} = 12 - 2y$

Squaring both sides

$(2\sqrt{y^2+13y+36})^2 = (12 - 2y)^2$

$4(y^2 + 13y + 36) = (12)^2 + (2y)^2 - 2(12)(2y)$

$4y^2 + 52y + 144 = 144 + 4y^2 - 48y$

$4y^2 + 52y + 144 - 144 - 4y^2 + 48y = 0$

$100y = 0$

$y = \frac{0}{100}$

$y = 0$

From (i)

$x^2 + 3x = y$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0 \qquad x + 3 = 0$

$x = -3$

Check:

Put $x = 0$

$\sqrt{(0)^2 + 3(0) + 9} + \sqrt{(0)^2 + 3(0) + 9} = 5$

$\sqrt{0+0+9} + \sqrt{0+0+4} = 5$

$\sqrt{9} + \sqrt{4} = 5$

$3 + 2 = 5$

$5 = 5$

Put $x = -3$

$\sqrt{(-3)^2 + 3(-3) + 9} + \sqrt{(3)^2 + 3(-3) + 4} = 5$

$\sqrt{9-9+9} + \sqrt{9-9+4} = 5$

$\sqrt{9} + \sqrt{4} = 5$

$3 + 2 = 5$

$5 = 5$

Solution set = $\{0, -3\}$

MISCELLANEOUS EXERCISE 1

Q.1 Multiple choice questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is:

(a) $bx + c = 0, b \neq 0$
(b) $ax^2 + bx + c = 0, a \neq 0$
(c) $ax^2 = bx, a \neq 0$ (d) $ax^2 = 0, a \neq 0$

(ii) The number of terms in a standard quadratic equations $ax^2 + bx + c = 0$ is:

(a) 1 (b) 2
(c) 3 (d) 4

(iii) The number of methods to solve a quadratic equation is:

(a) 1 (b) 2
(c) 3 (d) 4

(iv) The quadratic formula is:

(a) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

(b) $x = \frac{b \pm \sqrt{b^2-4ac}}{2a}$

(c) $x = \frac{-b \pm \sqrt{b^2+4ac}}{2a}$

(d) $x = \frac{b \pm \sqrt{b^2+4ac}}{2a}$

(v) Two linear factors of $x^2 - 15x + 56$ are:

(a) $(x - 7)$ and $(x + 8)$
(b) $(x + 7)$ and $(x - 8)$
(c) $(x - 7)$ and $(x - 8)$
(d) $(x + 7)$ and $(x + 8)$

(vi) An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an:

(a) Radical equation
(b) Reciprocal equation
(c) Exponential equation
(d) None of these

(vii) An equation of the type $3^x + 3^{2-x} + 6 = 0$ is called a/an:

(a) Reciprocal equation
(b) Radical equation
(c) Exponential equation
(d) None of these

(viii) the solution set of equation $4x^2 - 16 = 0$ is

(a) $\{\pm 4\}$ (b) $\{4\}$
(c) $\{\pm 2\}$ (d) ± 2

(ix) An equation of the form $2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$ is called a/an:

(a) Reciprocal equation
(b) Radical equation
(c) Exponential equation
(d) None of these

ANSWERS

(i)	b	(ii)	c	(iii)	c	(iv)	a	(v)	c
(vi)	b	(vii)	c	(viii)	c	(Ix)	a		

Q.2 Write short answers of the following questions:

(i) Solve $x^2 + 2x - 2 = 0$

Sol. We have

$x^2 + 2x - 2 = 0$ …… (A)

Compare it with $ax^2 + bx + c = 0$ we get

$a = 1, \quad b = 2, \quad c = -2,$

Putting the values in

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{4+8}}{2}$$

$$= \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm \sqrt{4 \times 3}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-1 \pm \sqrt{3})}{2} = -1 \pm \sqrt{3}$$

Solution set $= \{-1 \pm \sqrt{3}\}$

(ii) Solve by factorization $5x^2 = 15x$

Sol. We have

$5x^2 = 15x$

$5x^2 - 15x = 0 \Rightarrow 5x(x-3) = 0$

$5x = 0$ | $x - 3 = 0$

$x = \frac{0}{5}$ | $\Rightarrow x = 3$

$= 0$

Solution set $= \{0,3\}$

(iii) Write in standard form $\frac{1}{x+4} + \frac{1}{x-4} = 3$

Sol. We have

$$\frac{1}{x+4} + \frac{1}{x-4} = 3$$

$$\frac{x-4+x+4}{(x+4)(x-4)} = 3$$

$\Rightarrow$ $2x = 3(x+4)(x-4)$

$2x = 3(x^2 - 16)$

$2x = 3x^2 - 48$

$3x^2 - 2x - 48 = 0$ is standard form.

(iv) Write the names of the methods for solving a quadratic equation.

Sol. (i) Factorization
(ii) Completing square
(iii) Quadratic formula

(v) Solve $\left(2x - \frac{1}{2}\right)^2 = \frac{9}{4}$

Sol. We have

$$\left(2x - \frac{1}{2}\right)^2 = \frac{9}{4}$$

Taking square root on both sides

$$\sqrt{\left(2x - \frac{1}{2}\right)^2} = \sqrt{\frac{9}{4}}$$

$$2x - \frac{1}{2} = \pm\frac{3}{2}$$

$2x - \frac{1}{2} = \frac{3}{2}$ | $2x - \frac{1}{2} = -\frac{3}{2}$

$2x = \frac{1}{2} + \frac{3}{2}$ | $2x = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2}$

$2x = \frac{4}{2}$ | $x = \frac{-2}{2 \times 2}$

$x = \frac{4}{2 \times 2}$ | $x = \frac{-1}{2}$

$x = \frac{4}{4} = 1$

Solution set $= \left\{1, \frac{-1}{2}\right\}$

(vi) Solve $\sqrt{3x + 18} = x$

Sol. $\sqrt{3x + 18} = x$

Taking square on both sides

$$(\sqrt{3x+18})^2 = x^2$$

$$3x + 18 = x^2 \Rightarrow x^2 - 3x - 18 = 0$$

$$x^2 - 6x + 3x - 18 = 0$$

$$x(x-6)+3(x-6) = 0$$

$$(x-6)(x+3) = 0$$

$$x - 6 = 0 \quad x + 3 = 0$$

$$x = 6 \qquad x = -3$$

$$x = \{-3, 6\}$$

(vii) Define quadratic equation.

Sol. An equation which contains the square of the unknown (variable) quantity, but no higher power, is a quadratic equation or an equation of the second degree and standard form quadratic equation

$ax^2 + bx + c = 0$

(viii) Define reciprocal equation.

Sol. An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$. Standard form of reciprocal equation

$ax^4 + bx^3 + cx^2 + bz + a = 0$

(ix) Define exponential equation.

Sol. The equation in which variables occur in exponents is exponential equation.

(x) Define radical equation.

Sol. An equation involving expression under the radical sign is a radical equation.

Q.3 Fill in the blanks:

(i) The standard form of the quadratic equation is ________.

(ii) The number of methods to solve a quadratic equation are ________.

(iii) The name of the method to derive a quadratic formula is ________.

(iv) The solution of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is ________.

(v) The solution set of $25x^2 - 1 = 0$ is ________.

(vi) An equation of the form $2^{2x} - 3.2^x + 5 = 0$ is called a/an ________ equation.

(vii) The solution set of the equation $x^2 - 9 = 0$ is ________.

(viii) An equation of the type $x^4+x^3+x^2+x+1 = 0$ is called a/an ________ equation.

(ix) A root of an equation, which do not satisfy the equation is called ________root.

(x) An equation involving impression of the variable under ________ is called radical equation.

ANSWERS

(i) $(ax^2 + bx + c = 0)$

(ii) (3) (iii) Completing square

(iv) $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ (v) $\left\{\pm \frac{1}{5}\right\}$

(vi) Exponential (vii) {$\underline{+}$ 3}

(viii) Reciprocal (ix) Extraneous

(x) Radical sign

SUMMARY

- An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.
- A second degree equation in one variable x, $ax^2 + bx + c = 0$
  where $a \neq 0$ and a, b, c are real numbers, is called the general or standard form of a quadratic equation.
- An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$
- In exponential equations, variables occur in exponents.
- An equation involving expression under the radical sign is called a radical equation.
- Quadratic formula for $ax^2 + bx + c = 0$, $a \neq 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Any quadratic equation is solved by the following three methods.
- (i) Factorization
  (ii) Completing square
  (iii) Quadratic formula

ADDITIONAL MCQ's

1. Solution set of $5x^2 - 125 = 0$:
   (a) {5} (b) {10} (c) {−5} **(d) {±5}**