UNIT – 1

MATRICES AND DETERMINANTS

Unit Outlines

After studying this unit the students will be able to:

- A matrix with real entries and relate its rectangular layout (Formation) with real life.
- Rows and columns of a matrix.
  The order of a matrix.
- Equality of two matrices.
- Define and identify row matrix, column matrix, rectangular matrix, square matrix, zero/null matrix, identity matrix, scalar matrix, diagonal matrix, transpose of a matrix, symmetric and skew-symmetric matrices.
- Know whether the given matrices are conformable for addition/subtraction.
- Add and subtract matrices.
- Multiply a matrix by a real number.
- Verify commutative and associative laws under addition.
- Define additive identity of a matrix.
- Known whether the given matrices are conformable for multiplication.
- Multiply two (or three) matrices.
- Verify associative law under multiplication.
- Verify distributive laws.
- Show with the help of an example that commutative law under multiplication does not hold in general (i.e., $AB \neq BA$).
- Define multiplicative identity of a matrix.
- Verify the result $(AB)^t = B^tA^t$.
- Define the Determinant of a square matrix.
- Evaluate determinant of a matrix.
- Define singular and non-singular matrices.
- Define adjoint of a matrix.
- Find multiplicative inverse of a non-singular matrix A and verify that
- $AA^{-1} = I = A^{-1}A$ where I is the identity matrix.
- Use adjoint method to calculate inverse of a non-singular matrix.
- Verify the result $(AB)^{-1} = B^{-1}A^{-1}$
- Solve a system of two linear equations and related real life problems in two unknowns using
- Matrix inversion method,
- Cramer's rule.

Introduction of Matrix:

The idea of Matrix was given by **"Arthur Cayley"**, an English mathematician of **19th century**. Who first developed **"Theory of Matrices" in 1858**.

Matrix:

A rectangular array or a formation of collection of real numbers, say 0, 1, 2, 3, 4 and 9, such as; $\begin{matrix} 1 & 3 & 4 \\ 9 & 2 & 0 \end{matrix}$ and then enclosed by brackets [ ] is said to form a matrix $\begin{bmatrix} 1 & 3 & 4 \\ 9 & 2 & 0 \end{bmatrix}$.

Matrix Name:

Matrices are denoted conventionally by capital letters A, B, C… X, Y, Z etc of English Alphabets.

Row of a matrix:

In matrix, the entries presented in **horizontal** way are called rows.

$$A = \begin{bmatrix} a & b & c \\ l & m & b \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \end{matrix}$$

In above matrix A, R_1 and R_2 are two rows.

Columns of a matrix:

In matrix, the entries presented in **vertical** way are called columns.

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 9 \end{bmatrix}$$
$$\downarrow \;\; \downarrow$$
$$C_1 \;\; C_2$$

In above matrix A, C_1 and C_2 are two columns.

Order of a Matrix:

The number of rows and columns in a matrix specifies its order.

The order of a matrix is denoted by $m \times n$ or m-by-n.

Here; "m" represented the number of rows and "n" represented the number of columns.

m – by – n
↓ ↓
No. of rows No. of columns .

If a matrix C has two rows and 3 columns. The order of matrix is 2-by-3.

$$C = \begin{bmatrix} 1 & 3 & 4 \\ 9 & 2 & 0 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \end{matrix}$$
$$\downarrow \;\; \downarrow \;\; \downarrow$$
$$C_1 \;\; C_2 \;\; C_3$$

The order of matrix C is 2-by-3.

Equal Matrices:

Let A and B be two matrices; if

(i) The order of A = the order of B

(ii) Their corresponding entries are equal or same

Then A and B are Equal matrices

Equal matrices are denoted by A = B

$$A = \begin{bmatrix} 2 & 4 \\ 9 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2+2 \\ 10-1 & 1+1+1 \end{bmatrix}$$

Matrix A and B are equal because they have same order which is 2-by-2 and same corresponding elements so, A = B.

EXERCISE 1.1

Q.1: Find the order of the following matrices.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix},$$

$$\mathbf{C} = [2 \quad 4], \quad \mathbf{D} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix},$$

$$\mathbf{E} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}, \quad \mathbf{F} = [2]$$

$$\mathbf{G} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$$

A $= \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$

order of matrix = 2–by–2

B $= \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

Sol. $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

order of matrix = 2– by–2

C = [2 4]

Sol. C = [2 4]

order of matrix = 1–by–2

D $= \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

Sol. $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

order of matrix = 3–by–1

E $= \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Sol. $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Order of matrix = 3 – by –2.

F = [2]

Sol. F = [2]

Order of matrix =1 – by –1.

G $= \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Sol. $G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Order of matrix =3 – by –3.

H $= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Sol. $H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Order of matrix =2 – by –3.

Q.2: Which of the following matrices are equal.

A = [3], B = [3 5], C = [5 –2],

D = [5 3], $E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$,

$F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ $G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$,

$H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, **I = [3 3+2]**

$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$,

Sol. Since order of A and C same and corresponding elements are also same so A = C

A = [3]	**A = C**
B = [3 5]	**B = I**
C = [5 –2]	**C = A**
D = [5 3]	**Not equal to any matrix**
$E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$	**E = J= H**
$F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$	**F = G**
$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$	**G = F**
$H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$	**H = J = E**
I = [3 3+2] = [3 5]	**I = B**
$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$	**J = H = E**

Q.3: Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Sol. $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

According to the definition of equal matrices.

$a + c = 0$ ……(i)

$c - 1 = 3$ ……(ii)

$a + 2b = -7$ ……(iii)

$4d - 6 = 2d$ ……(iv)

By using the(ii) equation

$c - 1 = 3$

$c = 3+1$

$c = 4$

Put c=4 in (i) equation

$a + c = 0$

$a + 4 = 0$

$a = 0 - 4$

$a = -4$

Put the value of a= – 4 in (iii) equation

$a + 2b = -7$

$-4+2b = -7$

$2b = -7 + 4$

$2b = -3$

$b = -\frac{3}{2}$ or – 1.5

By using (iv) equation

$4d - 6 = 2d$

$4d - 2d = 6$

$2d = 6$

$d = \frac{6}{2}$

$d = 3$

Hence the value of

a = –4, b = –1.5, c = 4, d = 3

TYPES OF MATRICES

(i) Row Matrix:

A matrix is called a row matrix if it has only one row.

e.g. D = [1 3 4]

D is a row matrix and its order is 1-by-3.

(ii) Column Matrix:

A matrix is called a column matrix if it has only one column

e.g; $E = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 7 \end{bmatrix}$

E is a column matrix and its order is 4-by-1.

(iii) Rectangular Matrix:

A matrix A is called rectangular if, its number of rows is not equal to the number of its columns.

e.g; $A = \begin{bmatrix} 1 & 7 \\ 2 & 4 \\ 3 & 9 \end{bmatrix}$ $B = \begin{bmatrix} a & b & c \\ d & c & f \end{bmatrix}$

Order of A = 3-by-2

Order of B 2-by-3

(iv) Square Matrix:

A matrix is called a square matrix if its number of rows is equal to its number of columns.

e.g; C = [9] $D = \begin{bmatrix} 2 & 4 \\ 9 & 7 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 7 & 0 \\ 3 & 9 & 3 \\ 5 & 11 & 2 \end{bmatrix}$

order 1-by-1 order 2-by-2 order 3-by-3

(v) Null or Zero Matrix:

A matrix is called a null or zero matrix if each of its entries/elements are zero (0).

e.g; $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, O = [0 0 0], $O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

order 2-by-2 order 1-by-3 order 3-by-1

It is represented by O.

(vi) Transpose of a Matrix:

A matrix obtained by interchanging the row of matrix into the columns of that matrix.

OR

A matrix obtained by changing the columns into rows of a matrix.

If A is a matrix then transpose is denoted by A^t.

e.g; $A = \begin{bmatrix} 0 & 4 \\ 1 & 5 \\ 2 & 7 \end{bmatrix}$ then $A^t = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 7 \end{bmatrix}$

order = 3-by-2 then transpose order = 2-by-3

(vii) Negative of a Matrix:

Let A be a matrix. Then its negative is obtained by changing the signs of all the elements of A, i.e.

if $A = \begin{bmatrix} 2 & 9 \\ 4 & -3 \end{bmatrix}$, then $-A = \begin{bmatrix} -2 & -9 \\ -4 & 3 \end{bmatrix}$

(viii) Symmetric Matrix:

A square matrix is symmetric if it is equal to its transpose.

i.e; matrix A is symmetric if $A^t = A$.

$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$, $A^t = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A^t = A$

So, A is symmetric matrix.

(ix) Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric if $A^t = -A$.

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$$

$$\text{then } A^t = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -3 \\ -(-2) & 0 & -1 \\ -(-3) & -(-1) & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -A$$

Since $A^t = -A$, therefore A is skew-symmetric matrix.

(x) Diagonal Matrix:

A square matrix A is called a diagonal matrix if each element is zero except diagonal elements.

e.g; $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(xi) Scalar Matrix:

A diagonal matrix is called a scalar matrix, if all the diagonal elements are same and non-zero.

$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ $k \neq 0, 1$ $C = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

is a scalar matrix of order 3-by-3.

(xii) Identity Matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal elements are 1 and it is denoted by I.

e.g; $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I = [1]$

are all unit matrices,

Remember:

Note: (i) The scalar matrix and identity matrix are diagonal matrix.

(ii) Every diagonal matrix is not a scalar or identity matrix.

EXERCISE 1.2

Q.1: From the following matrices, identify unit matrices row matrices, column matrices and null matrices.

$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B} = [2 \quad 3 \quad 4]$,

$\mathbf{C} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mathbf{E} = [0]$, $\mathbf{F} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **is a null matrix.**

$B = [2 \quad 3 \quad 4]$ **is a row matrix.**

$C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$ **is a column matrix.**

$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **is a unit matrix.**

$E = [0]$ **is a null matrix.**

$F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$ **is a column matrix.**

Q.2: From the following matrices identify:
(a) Square matrices
(b) Rectangular matrices
(c) Row matrices
(d) Column matrices
(d) Identity matrices
(f) Null Matrices

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **(vi)** $[3\ 10\ -1]$

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. (i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ **Rectangular matrix**

(ii) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ **Column matrix, rectangular matrix**

(iii) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$ **Square matrix**

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **Identity matrix, square matrix**

(v) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **Rectangular matrix**

(vi) $[3\ \ 10\ \ -1]$ **Row matrix, rectangular matrix**

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **Column matrix, rectangular matrix**

(viii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Square matrix**

(ix) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ **is a null matrix, rectangular matrix**

Q.3: From the following matrices. Identify diagonal, scalar and unit (identity) matrices.

$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ **Scalar matrix, diagonal matrix**

$B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ **Diagonal matrix**

$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **Identity matrix, diagonal matrix**

$D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ **Diagonal matrix**

$E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$ $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ **Scalar matrix, diagonal matrix**

Q.4: Find negative of matrices A, B, C, D and E when

$A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$,

$D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$, $E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

A $= \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \Rightarrow -A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

B $= \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

Sol. $-B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

C $= \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$

Sol. $-C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$

D $= \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$

Sol. $-D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$

E $= \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

Sol. $-E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$

Q.5: Find the transpose of each of the following matrices.

$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $B = [5\ \ 1\ \ -6]$, $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Sol. $\mathbf{A} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

$\mathbf{A^t} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}^t$

$\mathbf{A^t} = [0 \quad 1 \quad -2]$

Sol. $\mathbf{B} = [5 \quad 1 \quad -6]$

$B^t = [5 \quad 1 \quad -6]^t$

$\mathbf{B^t} = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$

Sol. $\mathbf{C} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$

$\mathbf{C^t} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}^t$

$\mathbf{C^t} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$

Sol. $\mathbf{D} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$D^t = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}^t$

$\mathbf{D^t} = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

Sol. $\mathbf{E} = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

$E^t = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}^t$

$\mathbf{E^t} = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$

Sol. $\mathbf{F} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$F^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^t$

$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Q.6: Verify that if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ then (i) $(A^t)^t = A$ (ii) $(B^t)^t = B$

(i) $(A^t)^t = A$

Sol. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$

$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$(A^t)^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$

$\mathbf{(A^t)^t} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \mathbf{A}$

(ii) $(B^t)^t = B$

Sol. $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$

$(B^t) = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$(B^t)^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}^t$

$\mathbf{(B^t)^t} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = \mathbf{B}$

Addition and Subtraction of Matrices:

1. Addition of Matrices:

Let A and B be any two matrices with real entries; Matrices A and B are conformable for addition, if they have same order. Addition is denoted by A + B and is obtained by adding the entries of the matrix A to the corresponding entries of B.

$A = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix}$

$A+B = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2+3 & 3-2 & 0+5 \\ 5-1 & 6+4 & 1+1 \\ 2+4 & 1+2 & 3-4 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 5 \\ 4 & 10 & 2 \\ 6 & 3 & -1 \end{bmatrix}$

Or $B+A = \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3+2 & -2+3 & 5+0 \\ -1+5 & 4+6 & 1+1 \\ 4+2 & 2+1 & -4+3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 5 \\ 4 & 10 & 2 \\ 6 & 3 & -1 \end{bmatrix}$

2. Subtraction of Matrices:

Let A and B any two matrices. Matrices A and B are conformable for subtraction, if they have same order represented by A – B and is obtained by subtracting the entries of the matrix B to the corresponding entries of A.

$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 & 2 \\ -1 & 4 & 3 \end{bmatrix}$ are conformable for subtraction.

i.e. **A-B =** $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 2 \\ -1 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2-0 & 3-2 & 4-2 \\ 1-(-1) & 5-4 & 0-3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

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3. Commutative and Associative Law of Addition of Matrices:

A + B = B + A Commutative w.r.t +

A+(B+C) = (A+B)+C Associative w.r.t +

4. Additive Identity of a Matrix:

If A and B are two matrices of same order and

A + B = A or B + A = A

then matrix B is called additive identity of matrix A.

For any matrix A and zero matrix O of same order, O is called additive identity of A as

A + O = A = O + A

5. Additive Inverse of a Matrix:

If A and B are two matrices of same order and

A + B = 0 = B + A

Then A and B are called additive inverse of each other.

Note: Additive inverse of any matrix A is –A obtained by changing their signs of each element.

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad -A = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

EXERCISE 1.3

Q.1. Which of the following matrices are conformable for addition ?

$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$,

$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$, $E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ $F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ 2 – by – 2

$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ 2 – by – 1

$C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$ 3 – by – 2

$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ 2 – by 1

$E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ 2 – by – 2

$F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & -4 \\ 5 & 3 \end{bmatrix}$ 3 – by – 2

A and E are conformable for addition because their orders are same.

B and D are conformable for addition because their orders same.

C and F are conformable for addition because their orders are same.

Q.2: Find the additive inverse of following matrices.

$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$,

$C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$,

$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Sol. **A** $= \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

$-A = \begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$ is additive inverse of A

Sol. $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

$-B = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$ is additive inverse of B

Sol. $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$ is additive inverse of C

Sol. $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$ is additive inverse of D

$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

Sol. $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is additive inverse of E

Sol. $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$ is additive inverse of F

Q.3. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$C = [1 \quad -1 \quad 2]$, $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ then

find, (i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(iii) $C + [-2 \quad 1 \quad 3]$ (iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(v) 2A (vi) (–1) B (vii) (–2)C

(viii) 3D (ix) 3C

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Sol. $= A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Sol. $= B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 \\ -1+3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $C + [-2 \quad 1 \quad 3]$

Sol. $= C + [-2 \quad 1 \quad 3]$

$= [1 \quad -1 \quad 2] + [-2 \quad 1 \quad 3]$

$= [1-2 \quad -1+1 \quad 2+3]$

$= [-1 \quad 0 \quad 5]$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Sol. $= D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) 2A

Sol. = 2A

$= 2\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} (2)(-1) & (2)(2) \\ (2)(2) & (2)(1) \end{bmatrix}$

$2A = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) (–1)B

Sol. $= -1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-1)(1) \\ (-1)(-1) \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) (–2) C

Sol. $= -2[1 \quad -1 \quad 2]$

$= [(-2)(1) \quad (-2)(-1) \quad (-2)(2)]$

$= [-2 \quad 2 \quad -4]$

(viii) 3D

Sol. $= 3\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} (3)(1) & (3)(2) & (3)(3) \\ (3)(-1) & (3)(0) & (3)(2) \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

(ix) 3C

Sol. $= 3[1 \quad -1 \quad 2]$

$= [(3)(1) \quad (3)(-1) \quad (3)(2)]$

$\mathbf{= [3 \quad -3 \quad 6]}$

Q.4: Perform the indicated operations and simplify the following:

Two matrices are said to conformable for addition and subtraction if their orders same.

(i) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

Sol. $=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$=\begin{bmatrix}1+0 & 0+2\\0+3 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$=\begin{bmatrix}1+1 & 2+1\\3+1 & 1+0\end{bmatrix}$

$=\begin{bmatrix}\mathbf{2} & \mathbf{3}\\\mathbf{4} & \mathbf{1}\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

Sol. $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$=\begin{bmatrix}1+0 & 0+2\\0+3 & 1+0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$=\begin{bmatrix}1-1 & 2-1\\3-1 & 1-0\end{bmatrix}$

$=\begin{bmatrix}\mathbf{0} & \mathbf{1}\\\mathbf{2} & \mathbf{1}\end{bmatrix}$

(iii) $\mathbf{[2 \quad 3 \quad 1] + ([1 \quad 0 \quad 2] - [2 \quad 2 \quad 2])}$

Sol. $= [2 \quad 3 \quad 1] + ([1 \quad 0 \quad 2] - [2 \quad 2 \quad 2])$

$= [2 \quad 3 \quad 1] + ([1-2 \quad 0-2 \quad 2-2])$

$= [2 \quad 3 \quad 1] + [-1 \quad -2 \quad 0]$

$= [2-1 \quad 3-2 \quad 1+0]$

$= [1 \quad 1 \quad 1]$

(iv) $\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$

Sol. $=\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$

$=\begin{bmatrix}1+1 & 2+1 & 3+1\\-1+2 & -1+2 & -1+2\\0+3 & 1+3 & 2+3\end{bmatrix}$

$=\begin{bmatrix}2 & 3 & 4\\1 & 1 & 1\\3 & 4 & 5\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$

Sol. $=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$

$=\begin{bmatrix}1+1 & 2+0 & 3-2\\2-2 & 3-1 & 1+0\\3+0 & 1+2 & 2-1\end{bmatrix}$

$=\begin{bmatrix}\mathbf{2} & \mathbf{2} & \mathbf{1}\\\mathbf{0} & \mathbf{2} & \mathbf{1}\\\mathbf{3} & \mathbf{3} & \mathbf{1}\end{bmatrix}$

(vi) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

$=\begin{bmatrix}1+2+1 & 2+1+1\\0+1+1 & 1+0+1\end{bmatrix}$

$=\begin{bmatrix}\mathbf{4} & \mathbf{4}\\\mathbf{2} & \mathbf{2}\end{bmatrix}$

Q.5: For the matrices $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and** $C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$

verify the following rules:

(i) $\mathbf{A + C = C + A}$

Sol. L.H.S.

$A + C=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$

$\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$

R.H.S.

$C + A=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$

$C + A=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2+0\end{bmatrix}$

$C + A=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$

Hence proved

L.H.S = R.H.S

(ii) A + B = B + A

Sol. L.H.S

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$\mathbf{A + B} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{4} \\ \mathbf{4} & \mathbf{1} & \mathbf{3} \\ \mathbf{4} & \mathbf{0} & \mathbf{3} \end{bmatrix}$$

R.H.S

$$B + A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix}$$

$$\mathbf{B + A} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{4} \\ \mathbf{4} & \mathbf{1} & \mathbf{3} \\ \mathbf{4} & \mathbf{0} & \mathbf{3} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(iii) B + C = C + B

Sol. L.H.S

$$B + C = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$\mathbf{B + C} = \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{-4} & \mathbf{5} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \end{bmatrix}$$

R.H.S

$$C + B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$C + B = \begin{bmatrix} -1+1 & 0-1 & 0+1 \\ 0+2 & -2-2 & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix}$$

$$\mathbf{C + B} = \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{-4} & \mathbf{5} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(iv) A+(B + A) = 2A + B

Sol. L.H.S:

A + (B + A)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right)$$

A+ (B + A)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix} \right)$$

$$A + (B + A) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 & 3+4 \\ 2+4 & 3+1 & 1+3 \\ 1+4 & -1+0 & 0+3 \end{bmatrix}$$

$$A + (B + A) = \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

R.H.S:

$$2A + B = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

2A + B

$$= \begin{bmatrix} 2\times1 & 2\times2 & 2\times3 \\ 2\times2 & 2\times3 & 2\times1 \\ 2\times1 & 2\times-1 & 2\times0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$2A + B = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$2A + B = \begin{bmatrix} 2+1 & 4-1 & 6+1 \\ 4+2 & 6-2 & 2+2 \\ 2+3 & -2+1 & 0+3 \end{bmatrix}$$

$$2A + B = \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(v) (C – B) + A = C + (A – B)

Sol. L.H.S:

(C – B) + A

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

(C – B) + A

$$=\left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\ 0-2 & -2+2 & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

(C – B) + A

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

(C – B) + A

$$=\begin{bmatrix}-2+1 & 1+2 & -1+3\\ -2+2 & 0+3 & 1+1\\ -2+1 & 0-1 & -1+0\end{bmatrix}=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

R.H.S: C + (A – B)

C + (A – B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

C + (A – B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1-1 & 2+1 & 3-1\\ 2-2 & 3+2 & 1-2\\ 1-3 & -1-1 & 0-3\end{bmatrix}\right)$$

C + (A – B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$C+(A-B)=\begin{bmatrix}-1+0 & 0+3 & 0+2\\ 0+0 & -2+5 & 3-1\\ 1-2 & 1-2 & 2-3\end{bmatrix}$$

$$C+(A-B)=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

Hence proved.

L.H.S = R.H.S

(vi) 2A + B = A + (A + B)

Sol: L.H.S:

$$2A+B=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

2A + B

$$=\begin{bmatrix}(2)(1) & (2)(2) & (2)(3)\\ (2)(2) & (2)(3) & (2)(1)\\ (2)(1) & (2)(-1) & (2)(0)\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$2A+B=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$2A+B=\begin{bmatrix}2+1 & 4-1 & 6+1\\ 4+2 & 6-2 & 2+2\\ 2+3 & -2+1 & 0+3\end{bmatrix}$$

$$2A+B=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S:

A + (A + B)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

A + (A + B)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}\right)$$

$$A+(A+B)=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$A+(A+B)=\begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix}$$

$$A+(A+B)=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

Hence proved. L.H.S = R.H.S

(vii) (C – B) – A = (C – A) – B

Sol. L.H.S:

(C – B) – A

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

(C – B) – A

$$=\left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\ 0-2 & -2+2 & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix}\right)-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

(C – B) – A

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$(C-B)-A=\begin{bmatrix}-2-1 & 1-2 & -1-3\\ -2-2 & 0-3 & 1-1\\ -2-1 & 0+1 & -1-0\end{bmatrix}$$

$$(C-B)-A=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix}$$

R.H.S:

(C – A) – B

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right) - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

(C – A) – B

$$= \left(\begin{bmatrix} -1-1 & 0-2 & 0-3 \\ 0-2 & -2-3 & 3-1 \\ 1-1 & 1+1 & 2-0 \end{bmatrix} \right) - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

(C – A) – B

$$= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$(C - A) - B = \begin{bmatrix} -2-1 & -2+1 & -3-1 \\ -2-2 & -5+2 & 2-2 \\ 0-3 & 2-1 & 2-3 \end{bmatrix}$$

$$(C - A) - B = \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

(viii)(A + B) + C = A + (B + C)

Sol. L.H.S:

(A + B) + C

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

(A + B) + C

$$= \left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} \right) + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

(A + B) + C

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A + B) + C = \begin{bmatrix} 2-1 & 1+0 & 4+0 \\ 4+0 & 1-2 & 3+3 \\ 4+1 & 0+1 & 3+2 \end{bmatrix}$$

$$(A + B) + C = \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}$$

R.H.S:

A + (B + C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

A + (B + C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix} \right)$$

A + (B + C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 1+0 & 2-1 & 3+1 \\ 2+2 & 3-4 & 1+5 \\ 1+4 & -1+2 & 0+5 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}$$

Hence proved.

L.H.S = R.H.S

(ix) A + (B – C) = (A – C) + B

Sol. L.H.S:

A + (B – C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

A + (B – C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1+1 & -1-0 & 1-0 \\ 2-0 & -2+2 & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix} \right)$$

A + (B – C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$A + (B - C) = \begin{bmatrix} 1+2 & 2-1 & 3+1 \\ 2+2 & 3+0 & 1-1 \\ 1+2 & -1+0 & 0+1 \end{bmatrix}$$

$$A + (B - C) = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

R.H.S:

(A – C) + B

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

(A – C) + B

$$= \left(\begin{bmatrix} 1+1 & 2-0 & 3-0 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$(A-C)+B$

$$= \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$(A-C)+B = \begin{bmatrix} 2+1 & 2-1 & 3+1 \\ 2+2 & 5-2 & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix}$$

$$(A-C)+B = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

Hence proved.
L.H.S = R.H.S

(x) 2A + 2B = 2(A + B)

Sol. L.H.S:

$$2A + 2B = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$2A + 2B$

$$= \begin{bmatrix} (2)(1) & (2)(2) & (2)(3) \\ (2)(2) & (2)(3) & (2)(1) \\ (2)(1) & (2)(-1) & (2)(0) \end{bmatrix} + \begin{bmatrix} (2)(1) & (2)(-1) & (2)(1) \\ (2)(2) & (2)(-2) & (2)(2) \\ (2)(3) & (2)(1) & (2)(3) \end{bmatrix}$$

$$2A + 2B = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$2A + 2B = \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix}$$

$$2A + 2B = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}$$

R.H.S:

$2(A + B)$

$$= 2\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$2(A + B) = 2\left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} \right)$$

$$2(A + B) = 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$2(A + B) = \begin{bmatrix} 2(2) & 1(2) & 4(2) \\ 4(2) & 1(2) & 3(2) \\ 4(2) & 0(2) & 3(2) \end{bmatrix}$$

$$2(A + B) = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}$$

Hence proved. L.H.S = R.H.S

Q.6: If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$ find.

(i) 3A – 2B (ii) $2A^t - 3B^t$

(i) 3A – 2B

Sol.

$$3A - 2B = 3\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3(1) & 3(-2) \\ 3(3) & 3(4) \end{bmatrix} - \begin{bmatrix} 2(0) & 2(7) \\ 2(-3) & 2(8) \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix} = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

(ii) $2A^t - 3B^t$

Sol. $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix} \Rightarrow B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$$2A^t - 3B^t = 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$2A^t - 3B^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$2A^t - 3B^t = \begin{bmatrix} 2-0 & 6+9 \\ -4-21 & 8-24 \end{bmatrix}$$

$$\mathbf{2A^t - 3B^t = \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}}$$

Q.7. If $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$ then find a and b.

Sol.

$$2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

According to the definition of equal matrices

$8 + 3b = 10$ $\quad$ $2a - 12 = 1$ ……(ii)

$3b = 10 - 8$ $\quad$ $2a = 1 + 12$

$3b = 2$ $\quad$ $2a = 13$

$\mathbf{b = \frac{2}{3}}$ $\quad$ $\mathbf{a = \frac{13}{2}}$

Q.8: If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ then verify that

(i) $(A + B)^t = A^t + B^t$

L.H.S

$= (A + B)^t$

Sol.

$= (A+B)$

$$(A + B) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$(A + B) = \begin{bmatrix} 1+1 & 2+1 \\ 0+2 & 1+0 \end{bmatrix}$$

$$(A + B) = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

Now $(A + B)^t = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}^t$

$$(A + B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

R.H.S

$\therefore \quad A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+2 \\ 2+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

Hence proved. L.H.S = R.H.S

(ii) $(A - B)^t = A^t - B^t$

L.H.S

$(A - B)^t$

$$A - B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1-1 & 2-1 \\ 0-2 & 1-0 \end{bmatrix}$$

$$(A - B) = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$(A - B)^t = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$$

$$(A - B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

R.H.S

$= A^t - B^t$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(iii) $A + A^t$ is symmetric $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Sol. $A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

So proved according to the definition of symmetric matrix $(A + A^t) = (A + A^t)^t$

(iv) $A - A^t$ is a skew symmetric $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Sol. $A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A - A^t = \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$$

Now $-(A - A^t) = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$$(A - A^t)^t = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -(A - A^t)^t$$

So proved according to the definition of skew symmetric matrix $(A - A^t) = -(A - A^t)^t$

(v) $B + B^t$ is symmetric matrix $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Sol. $B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$B + B^t = \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$

$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$

$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} = -(B + B^t)$

So according to the definition of symmetric matrix, $B + B^t$ is symmetric matrix.

(vi) $B - B^t$ is symmetric $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Sol. $B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

To prove that $(B - B^t)$ is skew symmetric matrix of $(B - B^t)$

$(B - B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$

$(B - B^t)^t = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So $B - B^t$ is skew symmetric matrix.

EXERCISE 1.4

Note: Two Matrices A and B are conformable for multiplication (giving product AB) if number of columns of 1st Matrix (i.e. A) equal to the number of rows of 2nd Matrix (i.e. B).

Q.1: Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

No. of columns of 1st Matrix = 2
No. of rows of 2nd Matrix = 2

Sol. $2-by-2 \quad 2-by-1$

So it is possible

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

No. of columns of 1st Matrix = 2
No. of rows of 2nd Matrix = 2

Sol. $2-by-2 \quad 2-by-2$

So it is possible

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No. of columns of 1st Matrix = 1
No. of rows of 2nd Matrix = 2

Sol. $2-by-1 \quad 2-by-2$

So it is impossible

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

No. of columns of 1st Matrix = 2
No. of rows of 2nd Matrix = 2

Sol. 3–by–2 2–by–3

So it is possible

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

No. of columns of 1st Matrix = 3
No. of rows of 2nd Matrix = 3

Sol. $2-by-3 \quad 3-by-2$

So it is possible

Q.2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible).

(i) AB

Sol. $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$ product is possible.

So $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$AB = \begin{bmatrix} (3)(6) + (0)(5) \\ (-1)(6) + (2)(5) \end{bmatrix}$

$AB = \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix} \Rightarrow$ **$AB = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$**

(ii) BA

Sol. $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

BA is impossible. ∵ According to rule

No. of colums of 1st =1

No. of rows of 2nd = 2

So Impossible

Q.3: Find the following product.

(i) $[1 \quad 2]\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol. $= \overrightarrow{[1 \quad 2]}\begin{bmatrix} 4 \\ 0 \end{bmatrix}\downarrow$

= [(1)(4) + (2)(0)]

= [4 + 0]

= [4]

(ii) $[1 \quad 2]\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Sol. $= [\overrightarrow{1 \quad 2}]\begin{bmatrix} 5 \\ -4 \end{bmatrix}\downarrow$

$= [(1)(5) + (2)(-4)]$

$= [5 - 8]$

$= [-3]$

(iii) $[-3 \quad 0]\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol. $= [\overrightarrow{-3 \quad 0}]\begin{bmatrix} 4 \\ 0 \end{bmatrix}\downarrow$

$= [(-3)(4) + (0)(0)]$

$= [-12 + 0]$

$= [-12]$

(iv) $[6 \quad -0]\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol. $= [\overrightarrow{6 \quad -0}]\begin{bmatrix} 4 \\ 0 \end{bmatrix}\downarrow$

$= [(6)(4) + (-0)(0)]$

$= [24 - 0]$

$= [24]$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Sol. $= \begin{bmatrix} \overrightarrow{1 \quad 2} \\ -3 \quad 0 \\ 6 \quad -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}\downarrow$

$= \begin{bmatrix} (1)(4)+(2)(0) & (1)(5)+(2)(-4) \\ (-3)(4)+(0)(0) & (-3)(5)+(0)(-4) \\ (6)(4)+(-1)(0) & (6)(5)+(-1)(-4) \end{bmatrix}$

$= \begin{bmatrix} 4+0 & 5-8 \\ -12+0 & -15-0 \\ 24-0 & 30+4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$

Q.4: Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Sol. $= \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} (2)(2)+(3)(3) & (2)(-1)+(3)(0) \\ (1)(2)+(1)(3) & (1)(-1)+(1)(0) \\ (0)(2)+(-2)(3) & (0)(-1)+(-2)(0) \end{bmatrix}$

$= \begin{bmatrix} 4+9 & -2+0 \\ 2+3 & -1+0 \\ 0-6 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$

4. (b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

Sol. $= \begin{bmatrix} \overrightarrow{1 \quad 2 \quad 3} \\ \overrightarrow{4 \quad 5 \quad 6} \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\downarrow$

$= \begin{bmatrix} (1)(1)+(2)(3)+(3)(-1) & (1)(2)+(2)(4)+(3)(1) \\ (4)(1)+(5)(3)+(6)(-1) & (4)(2)+(5)(4)+(6)(1) \end{bmatrix}$

$= \begin{bmatrix} 1+6-3 & 2+8+3 \\ 4+15-6 & 8+20+6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$

4.(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Sol. $= \begin{bmatrix} \overrightarrow{1 \quad 2} \\ \overrightarrow{3 \quad 4} \\ \overrightarrow{-1 \quad 1} \end{bmatrix}\begin{bmatrix} 1\downarrow & 2\downarrow & 3\downarrow \\ 4 & 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(2)(4) & (1)(2)+(2)(5) & (1)(3)+(2)(6) \\ (3)(1)+(4)(4) & (3)(2)+(4)(5) & (3)(3)+(4)(6) \\ (-1)(1)+(1)(4) & (-1)(2)+(1)(5) & (-1)(3)+(1)(1) \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$

$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$

4. (d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & \frac{-5}{2} \\ -4 & 4 \end{bmatrix}$

Sol. $= \begin{bmatrix} \overrightarrow{8 \quad 5} \\ \overrightarrow{6 \quad 4} \end{bmatrix}\begin{bmatrix} 2 & \frac{-5}{2} \\ -4 & 4 \end{bmatrix}\downarrow$

$= \begin{bmatrix} (8)(2)+(5)(-4) & (8)(\frac{-5}{2})+(5)(4) \\ (6)(2)+(4)(-4) & (6)(\frac{-5}{2})+(4)(4) \end{bmatrix}$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{-4} & \mathbf{0} \\ \mathbf{-4} & \mathbf{1} \end{bmatrix}$$

4. (e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. $= \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (-1)(0)+(2)(0) & (-1)(0)+(2)(0) \\ (1)(0)+(3)(0) & (1)(0)+(3)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Q.5: Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ **and** $C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ **verify that:**

(i) AB = BA It is called commutative law w.r.t multiplication.

Note: In Matrices this law does not hold generally. i.e $AB \neq BA$

Sol. AB = BA

L.H.S

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1-9 & -2-15 \\ 2-0 & 4-0 \end{bmatrix}$$

$$AB = \begin{bmatrix} \mathbf{-10} & \mathbf{-17} \\ \mathbf{2} & \mathbf{4} \end{bmatrix}$$

R.H.S

$$BA = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} (1)(-1)+(2)(2) & (1)(3)+(2)(0) \\ (-3)(-1)+(-5)(2) & (-3)(3)+(-5)(0) \end{bmatrix}$$

$$BA = \begin{bmatrix} -1+4 & 3+0 \\ 3-10 & -9-0 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} \mathbf{3} & \mathbf{3} \\ \mathbf{-7} & \mathbf{-9} \end{bmatrix}$$

Hence proved $\mathbf{AB \neq BA}$

(ii) A(BC) = (AB)C

Sol. Associative law of Multiplication

A(BC) = (AB)C **L.H.S.**

Firstly

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$BC = \begin{bmatrix} (1)(2)+(2)(1) & (1)(1)+(2)(3) \\ (-3)(2)+(-5)(1) & (-3)(1)+(-5)(3) \end{bmatrix}$$

$$BC = \begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} (-1)(4)+(3)(-11) & (-1)(7)+(3)(-18) \\ (2)(4)+(0)(-11) & (2)(7)+(0)(-18) \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -4-33 & -7-54 \\ 8-0 & 14-0 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

R.H.S.

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1-9 & -2-15 \\ 2-0 & 4-0 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} (-10)(2)+(-17)(1) & (-10)(1)+(-17)(3) \\ (2)(2)+(4)(1) & (2)(1)+(4)(3) \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$\mathbf{(AB)C} = \begin{bmatrix} \mathbf{-37} & \mathbf{-61} \\ \mathbf{8} & \mathbf{14} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(iii) A(B + C) = AB + AC

Sol. Distributive Law of Multiplication over addition.

L.H.S.

A(B + C)

$$B + C = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$A(B+C)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$

$A(B+C)=\begin{bmatrix}(-1)(3)+(3)(-2) & (-1)(3)+(3)(-2)\\ (2)(3)+(0)(-2) & (2)(3)+(0)(-2)\end{bmatrix}$

$A(B+C)=\begin{bmatrix}-3-6 & -3-6\\ 6-0 & 6-0\end{bmatrix}$

$A(B+C)=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$

R.H.S

$AB=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

$AB=\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}$

$AB=\begin{bmatrix}-1-9 & -2-15\\ 2-0 & 4-0\end{bmatrix}=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$

$AC=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$

$AC=\begin{bmatrix}(-1)(2)+(3)(1) & (-1)(1)+(3)(3)\\ (2)(2)+(0)(1) & (2)(1)+(0)(3)\end{bmatrix}$

$AC=\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$

$AC=\begin{bmatrix}1 & 8\\ 2 & 4\end{bmatrix}$

$AB+AC=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 2 & 4\end{bmatrix}$

$AB+AC=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & 4+2\end{bmatrix}$

$AB+AC=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$

Hence proved L.H.S = R.H.S

(iv) A(B – C) = AB – AC

Sol. Distributive Law of Multiplication over Subtraction.

L.H.S.

A(B – C)

$B-C=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$

$B-C=\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$

$B-C=\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$

$A(B-C)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$

$A(B-C)=\begin{bmatrix}(-1)(-1)+(3)(-4) & (-1)(1)+(3)(-8)\\ (2)(-1)+(0)(-4) & (2)(1)+(0)(-8)\end{bmatrix}$

$A(B-C)=\begin{bmatrix}1-12 & -1-24\\ -2-0 & 2-0\end{bmatrix}$

$\mathbf{A(B-C)=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}}$

R.H.S:

AB – AC

$AB=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

$AB=\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}$

$AB=\begin{bmatrix}-1-9 & -2-15\\ 2-0 & 4-0\end{bmatrix}$

$AB=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$

$AC=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$

$AC=\begin{bmatrix}(-1)(2)+(3)(1) & (-1)(1)+(3)(3)\\ (2)(2)+(0)(1) & (2)(1)+(0)(3)\end{bmatrix}$

$AC=\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$

$AC=\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$

$AB-AC=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$

$AB-AC=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$

$\mathbf{AB-AC=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}}$

Hence proved L.H.S =R.H.S

Q.6: For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$, $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$ verify that:

(i) $(AB)^t=B^tA^t$ (ii) $(BC)^t=C^tB^t$

(i) $(AB)^t=B^tA^t$

L.H.S. $(AB)^t$

Sol. $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

$AB=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

$AB=\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}$

$AB=\begin{bmatrix}-1-9 & -2-15\\ 2-0 & 4-0\end{bmatrix}$

$$AB = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}^t$$

$$(AB)^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

R.H.S = B^tA^t

$$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix},\ A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix},$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix},\ A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$B^tA^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$B^tA^t = \begin{bmatrix} (1)(-1)+(-3)(3) & (1)(2)+(-3)(0) \\ (2)(-1)+(-5)(3) & (2)(2)+(-5)(0) \end{bmatrix}$$

$$B^tA^t = \begin{bmatrix} -1-9 & 2-0 \\ -2-15 & 4-0 \end{bmatrix}$$

$$\mathbf{B^tA^t} = \begin{bmatrix} \mathbf{-10} & \mathbf{2} \\ \mathbf{-17} & \mathbf{4} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(ii) $(BC)^t = C^tB^t$

Sol. $(BC)^t = C^tB^t$

L.H.S

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$BC = \begin{bmatrix} (1)(-2)+(2)(3) & (1)(6)+(2)(-9) \\ (-3)(-2)+(-5)(3) & (-3)(6)+(-5)(-9) \end{bmatrix}$$

$$BC = \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}$$

$$(BC)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$$

$$\mathbf{(BC)^t} = \begin{bmatrix} \mathbf{4} & \mathbf{-9} \\ \mathbf{-12} & \mathbf{27} \end{bmatrix}$$

R.H.S

C^tB^t

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix},\ B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$C^tB^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$C^tB^t = \begin{bmatrix} (-2)(1)+(3)(2) & (-2)(-3)+(3)(-5) \\ (6)(1)+(-9)(2) & (6)(-3)+(-9)(-5) \end{bmatrix}$$

$$C^tB^t = \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$$

$$\mathbf{C^tB^t} = \begin{bmatrix} \mathbf{4} & \mathbf{-9} \\ \mathbf{-12} & \mathbf{27} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

Determinant:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by- 2 square matrix.

The determinant of A is denoted by det A or $|A|$ is defined as

$$|A| = \det A = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - cb = \lambda \in R$$

Example:

$$\mathbf{B} = \begin{bmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{-2} \end{bmatrix}$$

Sol. $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

$$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = (1)(-2) - (2)(3)$$

$$= -2 - 6 = -8$$

Singular matrix:

A square matrix is singular if its determinant is zero.e.g; see $|A| = 0$

$$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = (3)(4) - (2)(6)$$

$$= 12 - 12 = 0$$

Non-singular matrix:

A square matrix is non-singular if its determinant is not equal to zero. e.g; See $|C| \neq 0$

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix} = (7)(5) - (3)(-9)$$

$= 35 + 27 = 62 \neq 0$ C is a non-singular matrix.

Adjoint of a matrix:

Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the signs of other entries Ad joint of matrix A is denoted as Adj A

$$\text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

EXERCISE 1.5

Q.1: Find the determinant of the following matrices.

Note: A determinant is denoted by det A or |A|

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Sol. $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$|A| = (-1)(0) - (2)(1)$

$|A| = 0 - 2 = \mathbf{-2}$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Sol. $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$

$|B| = (1)(-2) - (2)(3)$

$|B| = -2 - 6 = \mathbf{-8}$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Sol. $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$|C| = (3)(2) - (3)(2)$

$|C| = 6 - 6 = \mathbf{0}$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Sol. $|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

$|D| = (3)(4) - (1)(2)$

$|D| = 12 - 2 = \mathbf{10}$

Q.2: Find which of the following matrices are singular or non-singular?

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

$|A| = (3)(4) - (2)(6)$

$|A| = 12 - 12 = \mathbf{0}$

A is a singular matrix because its determinant = 0

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Sol. $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$

$|B| = (4)(2) - (3)(1)$

$|B| = 8 - 3 \neq \mathbf{0}$

B is a non singular matrix because its determinant is $\neq 0$

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Sol. $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$

$|C| = (7)(5) - (3)(-9)$

$|C| = 35 + 27 = \mathbf{62}$

C is a non-singular matrix because its determinant is $\neq 0$

(iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Sol. $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$

$|D| = (5)(4) - (-2)(-10)$

$|D| = 20 - 20 = \mathbf{0}$

D is a singular matrix because its determinant=0

Q.3: Find the multiplicative inverse (if it exists) of each.

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

Sol. $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$

$|A| = (-1)(0) - (2)(3)$

$|A| = 0 - 6 = -6 \neq \mathbf{0}$

A is a Non-singular matrix so solution possible.

$\text{Adj } A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}}{-6}$$

$$A^{-1} = -\frac{1}{6}\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} (-\frac{1}{6})(0) & (-\frac{1}{6})(-3) \\ (-\frac{1}{6})(-2) & (-\frac{1}{6})(-1) \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) $\mathbf{B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}}$

Sol. $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$

$|B| = (1)(-5) - (-3)(2)$

$|B| = -5 + 6$

$= 1 \neq \mathbf{0}$

B is a Non singular matrix so solution possible.

$\text{Adj } B = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$\mathbf{B^{-1} = \frac{Adj\ B}{|B|}}$

$$B^{-1} = \frac{\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}}{1}$$

$$\mathbf{B^{-1} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}}$$

(iii) $\mathbf{C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}}$

Sol. $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$

$|C| = (-2)(-9) - (3)(6)$

$|C| = 18 - 18 = 0$

C is singular matrix so multiplicative inverse does not exists.

(iv) $\mathbf{D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}}$

Sol. $D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

$|D| \quad D = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$

$D` = \left(\frac{1}{2}\right)(2) - (1)\left(\frac{3}{4}\right)$

$D = 1 - \frac{3}{4}$

$D = \frac{4-3}{4} = \frac{1}{4} \neq \mathbf{0}$

D is a Non singular matrix so solution possible.

$D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

$\text{Adj } D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

$\mathbf{D^{-1} = \frac{Adj\ D}{|D|}}$

$$D^{-1} = \frac{\begin{vmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{vmatrix}}{\frac{1}{4}}$$

$$D^{-1} = 4\begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} (4)(2) & (4)(\frac{-3}{4}) \\ (-1)(4) & (\frac{1}{2})(4) \end{bmatrix}$$

$$\mathbf{D^{-1} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}}$$

Q.4: If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ then

(i) A(Adj A) = (Adj A)A = (det A)I

(ii) $BB^{-1} = I = B^{-1}B$

(i) A(Adj A) = (Adj A)A = (det A)I

Sol. $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} (1)(6)+(2)(-4) & (1)(-2)+(2)(1) \\ (4)(6)+(6)(-4) & (4)(-2)+(6)(1) \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

(Adj A)A = A(Adj A)

$A(\text{Adj } A) = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} (6)(1)+(-2)(4) & (6)(2)+(-2)(6) \\ (-4)(1)+(1)(4) & (-4)(2)+(1)(6) \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$

$A(\text{Adj } A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

(det A) I $\quad \therefore \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(\det A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$

$|A| = (1)(6) - (4)(2)$

$|A| = 6 - 8 = -2$

$(\det A)I = -2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(\det A)I = \begin{bmatrix} (-2)(1) & (-2)(0) \\ (-2)(0) & (-2)(1) \end{bmatrix}$

$(\det A)I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

It is proved A(Adj A) = (Adj A)A = (det A)I

(ii) $BB^{-1} = I = B^{-1}B$

Sol.

R.H.S $\quad B^{-1}B = I$

$B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$

$= (3)(-2) - (2)(-1) = -6 + 2 = -4 \neq 0$

B is a Non-singular matrix so solution is possible.

$\text{Adj } B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \text{ Adj } B$

$B^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$B^{-1}B = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$B^{-1}B = \frac{1}{-4}\begin{bmatrix} (-2)(3)+(1)(2) & (-2)(-1)+(1)(-2) \\ (-2)(3)+(3)(2) & (-2)(-1)+(3)(-2) \end{bmatrix}$

$B^{-1}B = \frac{1}{-4}\begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$

$B^{-1}B = \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$B^{-1}B = \begin{bmatrix} \frac{-4}{-4} & \frac{0}{-4} \\ \frac{0}{-4} & \frac{-4}{-4} \end{bmatrix}$

$B^{-1}B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I = B^{-1}B$

L.H.S

$BB^{-1} = \frac{1}{-4}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$BB^{-1} = \frac{1}{-4}\begin{bmatrix} (3)(-2)+(-1)(-2) & (3)(1)+(-1)(3) \\ (2)(-2)+(-2)(-2) & (2)(1)+(-2)(3) \end{bmatrix}$

$BB^{-1} = \frac{1}{-4}\begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$

$BB^{-1} = \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$BB^{-1} = \begin{bmatrix} \frac{-4}{-4} & \frac{0}{-4} \\ \frac{0}{-4} & \frac{-4}{-4} \end{bmatrix}$

$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence proved $BB^{-1} = I = B^{-1}B$

Q.5: Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ **and** $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Sol. $= \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} (3)(7)+(5)(-4) & (3)(-5)+(5)(3) \\ (4)(7)+(7)(-4) & (4)(-5)+(7)(3) \end{bmatrix}$

$= \begin{bmatrix} 21-20 & -15+15 \\ 28-28 & -20+21 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

So both matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ **and** $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Sol. $= \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1)(-3)+(2)(2) & (1)(2)+(2)(-1) \\ (2)(-3)+(3)(2) & (2)(2)+(3)(-1) \end{bmatrix}$

$= \begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow I$

So both matrices are multiplicative inverse of each other.

Q.6: If $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$,

$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ **then verify that**

(i) $(AB)^{-1} = B^{-1}A^{-1}$

(ii) $(DA)^{-1} = A^{-1}D^{-1}$

(i) $(AB)^{-1} = B^{-1}A^{-1}$

Sol.

L.H.S

$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix} =$

$AB = \begin{bmatrix} (4)(-4)+(0)(1) & (4)(-2)+(0)(-1) \\ (-1)(-4)+(2)(1) & (-1)(-2)+(2)(-1) \end{bmatrix}$

$AB = \begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2-2 \end{bmatrix} = \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$

$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$

$AB = (-16)(0) - (6)(-8) = 0 + 48$

$|AB| = 48$

$\text{Adj } AB = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

$(AB)^{-1} = \dfrac{\text{Adj } AB}{|AB|}$

$(AB)^{-1} = \dfrac{\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}}{48}$

$(AB)^{-1} = \dfrac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix} (\frac{1}{48})(0) & (\frac{1}{48})(8) \\ (\frac{1}{48})(-6) & (\frac{1}{48})(-16) \end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix} 0 & \frac{1}{6} \\ \frac{-1}{8} & \frac{-1}{3} \end{bmatrix}$

R.H.S

$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$|A| = (4)(2) - (-1)(0)$

$|A| = 8 - 0$

$|A| = 8$

$\text{Adj } A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$A^{-1} = \dfrac{\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}}{8} = \dfrac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} (2)(\frac{1}{8}) & (0)(\frac{1}{8}) \\ (1)(\frac{1}{8}) & (4)(\frac{1}{8}) \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$

$B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$

$|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$

$= (-4)(-1) - (1)(-2)$

$|B| = 4 + 2 = 6$

$\text{Adj } B = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$

$B^{-1} = \dfrac{\text{Adj } B}{|B|}$

$$B^{-1} = \frac{\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}}{6}$$

$$B^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} (\frac{1}{6})(-1) & (2)(\frac{1}{6}) \\ (\frac{1}{6})(-1) & (-4)(\frac{1}{6}) \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} (-\frac{1}{6})(\frac{1}{4})+(\frac{1}{3})(\frac{1}{8}) & (-\frac{1}{6})(0)+(\frac{1}{3})(\frac{1}{2}) \\ (-\frac{1}{6})(\frac{1}{4})+(-\frac{2}{3})(\frac{1}{8}) & (-\frac{1}{6})(0)+(-\frac{2}{3})(\frac{1}{2}) \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{24}+\frac{1}{24} & 0+\frac{1}{6} \\ -\frac{1}{24}-\frac{2}{24} & 0-\frac{2}{6} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 0 & \frac{1}{6} \\ \frac{-3}{24} & \frac{-1}{3} \end{bmatrix} \Rightarrow B^{-1}A^{-1} = \begin{bmatrix} 0 & \frac{1}{6} \\ \frac{-1}{8} & \frac{-1}{3} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

(ii) $\mathbf{(DA)^{-1} = A^{-1}D^{-1}}$

$$\mathbf{D} = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

Sol.

L.H.S

$$(DA) = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$(DA) = \begin{bmatrix} (3)(4)+(1)(-1) & (3)(0)+(1)(2) \\ (-2)(4)+(2)(-1) & (-2)(0)+(2)(2) \end{bmatrix}$$

$$(DA) = \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0+4 \end{bmatrix}$$

$$(DA) = \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$$

$|DA| = (11)(4) - (-10)(2)$

$= 44 + 20$

$|DA| = 64$

$$\text{Adj } DA = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$\mathbf{(DA)^{-1} = \frac{(Adj\ DA)}{|DA|}}$$

$$(DA)^{-1} = \frac{\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}}{64}$$

$$(DA)^{-1} = \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$(DA)^{-1} = \begin{bmatrix} (\frac{1}{64})(4) & (\frac{1}{64})(-2) \\ (\frac{1}{64})(10) & (\frac{1}{64})(11) \end{bmatrix}$$

$$(DA)^{-1} = \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

R.H.S:

$\mathbf{A^{-1}D^{-1}}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$|A| = (4)(2) - (-1)(0)$

$|A| = 8 - 0 = 8$

$$\text{Adj } A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\mathbf{A^{-1} = \frac{Adj\ A}{|A|}}$$

$$A^{-1} = \frac{\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}}{8}$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} (\frac{1}{8})(2) & (\frac{1}{8})(0) \\ (\frac{1}{8})(1) & (\frac{1}{8})(4) \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$$

$|D| = (3)(2) - (-2)(1)$

$|D| = 6 + 2 = 8$

$$\text{Adj}D = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\mathbf{D^{-1} = \frac{Adj\ D}{|D|}}$$

$$D^{-1} = \frac{\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}}{8}$$

$$D^{-1} = \begin{bmatrix} (\frac{1}{8})(2) & (\frac{1}{8})(-1) \\ (\frac{1}{8})(2) & (\frac{1}{8})(3) \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$A^{-1}D^{-1} = \begin{bmatrix} (\frac{1}{4})(\frac{1}{4})+(0)(\frac{1}{4}) & (\frac{1}{4})(-\frac{1}{8})+(0)(\frac{3}{8}) \\ (\frac{1}{8})(\frac{1}{4})+(\frac{1}{2})(\frac{1}{4}) & (\frac{1}{8})(-\frac{1}{8})+(\frac{1}{2})(\frac{3}{8}) \end{bmatrix}$$

$$A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{16}+0 & \frac{-1}{32}+0 \\ \frac{1}{32}+\frac{1}{8} & \frac{-1}{64}+\frac{3}{16} \end{bmatrix}$$

$$A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{1+4}{32} & \frac{-1+12}{64} \end{bmatrix} \quad A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

Hence proved L.H.S = R.H.S

Simultaneous equations:

A system of equation having a common solution is called a system of simultaneous equations

ax+by = m

cx+dy = n

where a,b,c,d,m and n are real number

x and y are two variable so these are two variable linear equations

EXERCISE 1.6

Q.1: Use matrices, if possible, to solve the following systems of linear equations by:

(i) The matrix inverse method

(ii) The Cramer's rule.

(i) $2x - 2y = 4$

$3x + 2y = 6$

Sol. Matrix inverse method

By writing in matrix form

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \ldots \text{(i)}$$

A X = B

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$|A| = (2)(2) - (3)(-2)$

$|A| = 4 + 6$

$|A| = 10 \neq 0$

Non-singular matrix so solution is possible

$$\text{Adj } A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

$X = A^{-1}B$

$$X = \frac{1}{|A|} \text{ Adj } A \times B$$

$$X = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} (2)(4) + (2)(6) \\ (-3)(4) + (2)(6) \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

x = 2,y = 0

(i) By Cramer's rule

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$= (2)(2) - (3)(-2)$

$= 4 + 6$

$= 10 \neq 0$

Non-singular matrix so solution is possible

$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$

$x = \dfrac{|A_x|}{|A|}$

$x = \dfrac{\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}}{10}$

$x = \dfrac{(4)(2)-(6)(-2)}{10}$

$x = \dfrac{8+12}{10}$

$x = \dfrac{20}{10} = 2$

$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

$y = \dfrac{|A_y|}{|A|}$

$y = \dfrac{\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}}{10}$

$y = \dfrac{(2)(6)-(3)(4)}{10}$

$y = \dfrac{12-12}{10}$

$y = \dfrac{0}{10} = 0$

x = 2, y = 0

(ii) **2x + y = 3**
6x + 5y = 1

Sol. **By matrix inverse method**:
Writing in the matrix form

$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. . ..(i)

A X = B

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$

$|A| = (2)(5) - (6)(1)$

$|A| = 10 - 6$

$|A| = 4 \neq 0$

Non-singular matrix so solution is possible

$\text{Adj } A = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$

$X = A^{-1}B$

$X = \dfrac{1}{|A|} \text{ Adj } A \times B$

$X = \dfrac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$X = \dfrac{1}{4}\begin{bmatrix} (5)(3)+(-1)(1) \\ (-6)(3)+(2)(1) \end{bmatrix}$

$X = \dfrac{1}{4}\begin{bmatrix} 15-1 \\ -18+2 \end{bmatrix}$

$X = \dfrac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$

$X = \begin{bmatrix} \dfrac{14}{4} \\ \dfrac{-16}{4} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{7}{2} \\ -4 \end{bmatrix}$

$\mathbf{x = \dfrac{7}{2}}$ **, y = – 4**

By Cramer's rule

$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. . . . .(i)

A X = B

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$

$|A| = (2)(5) - (6)(1)$

$|A| = 10 - 6$

$|A| = 4 \neq 0$

Non-singular matrix so solution is possible

$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$

$x = \dfrac{|A_x|}{|A|} = \dfrac{\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}}{|A|} = \dfrac{(3)(5)-(1)(1)}{4}$

$x = \dfrac{(3)(5)-(1)(1)}{4}$

$x = \dfrac{15-1}{4} = \dfrac{14}{4}$

$x = \dfrac{7}{2}$

$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$

$y = \dfrac{|A_y|}{|A|} = \dfrac{\begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}}{|A|}$

$y = \dfrac{(2)(1)-(3)(6)}{4} = \dfrac{2-18}{4} = \dfrac{-16}{4} = -4$

$$y \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$\mathbf{x = \frac{7}{2}\ ,\ y = -4}$$

(iii) $\mathbf{4x + 2y = 8}$

$\mathbf{3x - y = -1}$

Sol. By inverse method:

Writing in the matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$|A| = (4)(-1) - (3)(2)$

$|A| = -4 - 6$

$|A| = -10 \neq 0$

Non-singular matrix so solution is possible

$X = A^{-1} B$

$X = \frac{1}{|A|} \text{ Adj } A \times B$

$$\text{Adj } A = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

$$X = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$X = \frac{1}{-10}\begin{bmatrix} (-1)(8) + (-2)(-1) \\ -3(8) + (4)(-1) \end{bmatrix}$$

$$X = \frac{1}{-10}\begin{bmatrix} -8 + 2 \\ -24 - 4 \end{bmatrix}$$

$$X = \frac{1}{-10}$$

$$X = \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix} \qquad X = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix} -$$

$$x = \frac{3}{5}, y = \frac{14}{5}$$

Cramer's rule

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$|A| = (4)(-1) - (3)(2)$

$|A| = -4 - 6$

$|A| = -10 \neq 0$

Non singular matrix so solution is possible

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}}{-10}$$

$$X = \frac{(8)(-1)-(-1)(2)}{-10}$$

$$X = \frac{-8 + 2}{-10}$$

$$X = \frac{-6}{-10}$$

$$X = \frac{3}{5}$$

$$y = \frac{|A_y|}{|A|}$$

$$Y = \frac{\begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}}{-10}$$

$$y = \frac{(4)(-1) - (3)(8)}{-10}$$

$$y = \frac{-4 - 24}{-10} = \frac{-28}{-10}$$

$$y = \frac{14}{5}$$

$$\mathbf{x = \frac{3}{5}, y = \frac{14}{5}}$$

(iv) $\mathbf{3x - 2y = -6}$

$\mathbf{5x - 2y = -10}$

Sol. By inverse method

Writing in the matrix form

$$= \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$|A| = (3)(-2) - (-2)(5)$

$|A| = -6 + 10 = 4 \neq 0$

$X = A^{-1}B$

$X = \frac{1}{|A|}\text{ Adj } A \times B$

$X = \frac{1}{4}\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} (-2)(-6)+(2)(-10) \\ (-5)(-6)+(3)(-10) \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 12-20 \\ 30-30 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} -8 \\ 0 \end{bmatrix}$

$X = \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$x = -2, y = 0$

Cramer's Rule

$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$

$|A| = (3)(-2) - (5)(-2) \neq 0$

$|A| = -6 + 10 = 4$

Non singular matrix so solution possible

$x = \frac{|Ax|}{|A|}$

$x = \frac{\begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}}{4}$

$x = \frac{(-6)(-2) - (-10)(-2)}{4}$

$x = \frac{12-20}{4}$

$x = \frac{-8}{4}$

$x = -2$

$y = \left|\frac{Ay}{A}\right|$

$y = \frac{\begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}}{4}$

$y = \frac{(3)(-10)-(5)(-6)}{4}$

$= \frac{-30+30}{4}$

$= \frac{0}{4} = 0$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ **x= –2, y = 0**

(v) 3x – 2y = 4

–6x + 4y = 7

Sol. By inverse method:

Writing in the matrix form

$\begin{bmatrix} 3 & -2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$

$=(3)(4) - (-6)(-2)$

$= 12-12=0$

It is singular matrix so solution set is impossible

(vi) 4x + y = 9

–3x – y = –5

Writing in the matrix form

Sol. By inverse method

$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1) - (-3)(1)$

$= -4 + 3 = -1 \neq 0$ Non singular so possible

$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

$X = \frac{1}{|A|} Adj\ A \times B$

$X = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$X = \frac{1}{-1}\begin{bmatrix} (-1)(9)+(-1)(-5) \\ (3)(9)\ +(4)(-5) \end{bmatrix}$

$X = \frac{1}{-1}\begin{bmatrix} -9+5 \\ 27-20 \end{bmatrix}$

$X = \frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$

$X = \begin{bmatrix} \frac{-4}{1} \\ \frac{7}{-1} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

x = 4 , y = –7

Cramer's Rule

$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1) - (-3)(1)$

$= -4 + 3 = -1$

$x = \frac{|Ax|}{|A|}$

$= \frac{\begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}}{-1}$

$= \frac{(9)(-1)-(-5)(1)}{-1}$

$= \frac{-9+5}{-1}$

$= \frac{-4}{-1}$

x = 4

$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}}{-1}$

$= \frac{(4)(-5)-(-3)(9)}{-1}$

$= \frac{-20+27}{-1} \qquad = \frac{7}{-1}$

$= -7$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

x = 4 , y = –7

(vii) 2x – 2y = 4

–5x – 2y = –10

Writing in the matrix form

Sol. By inverse matrix

$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14$

Non singular matrix so solution is possible

$Adj\ A = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

$X = A^{-1}B$

$X = \frac{1}{|A|}\ Adj\ A \times B$

$= \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$x = \frac{1}{-14}\begin{bmatrix} (-2)(4)+(2)(-10) \\ (5)(4)+(2)(-10) \end{bmatrix}$

$X = \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$

$X = \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$

$X = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{-14} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

x= 2, y = 0

Crammer's Rule

$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

A X = B

$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14 \neq 0$

Non singular matrix so solution possible

$$x = \frac{Ax}{A}$$

$$= \frac{\begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}}{-14}$$

$$= \frac{(4)(-2)-(-10)(-2)}{-14}$$

$$= \frac{-8-20}{-14}$$

$$\frac{-28}{-14} = 2$$

$$y = \frac{|Ay|}{|A|} = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}$$

$$= \frac{(2)(-10)-(-5)(4)}{-14} \quad y = 0$$

$$= \frac{-20+20}{-14}$$

$x = 2$, $y = 0$

(viii) 3x – 4y = 4

x + 2y = 8

Sol. By inverse method

Writing in the matrix form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$A \quad X = B$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = (3)(2) - (-4)(1) = 6 + 4$$

$= 10 \neq 0$ Non singular so possible

$$\text{Adj } A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} Adj\ A \times B$$

$$X = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} (2)(4)+(4)(8) \\ (-1)(4)+(3)(8) \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$$

$$X = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

{x = 4, y = 2}

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Crammer's Rule

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = (3)(2)-(1)(-4) = 6+4 = 10$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}}{10}$$

$$= \frac{(4)(2)-(8)(-4)}{10}$$

$$= \frac{8+32}{10}$$

$$= \frac{\cancel{40}^{4}}{\cancel{10}_{1}}$$

$=4$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}}{10}$$

$$= \frac{(3)(8)-(1)(4)}{10} = \frac{20}{10}$$

$y = 2$

2: The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Sol. Suppose lengths of rectangle

Width= x cm

Length= y cm

According to given condition

4x = y

4x – y = 0

2x + 2y=150

Writing the in matrix form

$$\begin{bmatrix}4 & -1\\2 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\150\end{bmatrix}$$

A X = B

Where $A=\begin{bmatrix}4 & -1\\2 & 2\end{bmatrix}$, X= $\begin{bmatrix}x\\y\end{bmatrix}$, $B=\begin{bmatrix}0\\150\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & -1\\2 & 2\end{vmatrix}$$

=(4)(2) – (2)(–1) = 8 + 2 = 10

$$\text{Adj A} = \begin{bmatrix}2 & 1\\-2 & 4\end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj A}}{|A|}$$

$$A^{-1}=\frac{\begin{bmatrix}2 & 1\\-2 & 4\end{bmatrix}}{10}$$

$$=\begin{bmatrix}\frac{2}{10} & \frac{1}{10}\\\frac{-2}{10} & \frac{4}{10}\end{bmatrix}$$

$X=A^{-1}B$

$$\begin{bmatrix}\frac{2}{10} & \frac{1}{10}\\\frac{-2}{10} & \frac{4}{10}\end{bmatrix}\begin{bmatrix}0\\150\end{bmatrix}$$

$$=\begin{bmatrix}\left(\frac{2}{10}\right)(0)+\left(\frac{1}{10}\right)(150)\\\left(\frac{-2}{10}\right)(0)+\left(\frac{4}{10}\right)(150)\end{bmatrix}$$

$$=\begin{bmatrix}0 & 15\\0 & 60\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}15\\60\end{bmatrix}$$

x = 15 y = 60

Thus Width = 15cm Length = 60cm

Cramer's Rule

$$A_x=\begin{bmatrix}0 & -1\\150 & 2\end{bmatrix}$$

$$A_y=\begin{bmatrix}4 & 0\\2 & 150\end{bmatrix}$$

$$|A| = \begin{vmatrix}4 & -1\\2 & 2\end{vmatrix}$$

= (4)(2) – (2)(–1) = 8 + 2 = 10 $\neq 0$

$$x=\left|\frac{A_x}{A}\right|$$

$$=\frac{(0)(2)\text{-}(150)(\text{ -}1)}{10}$$

$$=\frac{0+150}{10}=\frac{150}{10}$$

X = 15

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{\begin{vmatrix}4 & 0\\2 & 150\end{vmatrix}}{10}$$

$$=\frac{(4)(150)\text{-}(2)(0)}{10}=\frac{600}{10}$$

= 60

x = 15

y = 60

3: Two sides of a rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.

Sol. Suppose

Length of a rectangle =x cm

Width of a rectangle =y cm

According to the condition

x –y = 3.5

2x + 2y = 67

Writing the in matrix form

$$\begin{bmatrix}1 & -1\\2 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3.5\\67\end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix}1 & -1\\2 & 2\end{vmatrix}=(1)(2)-(2)(-1)=2+2=4$$

$$\text{Adj } A = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}}{4}$$

$$= \begin{bmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{1}{4} \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \begin{bmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{4}(3.5) + \frac{1}{4}(67) \\ \frac{-2}{4}(3.5) + \frac{1}{4}(67) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{0} + \frac{67}{4} \\ \frac{7}{4} + \frac{67}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{74}{4} \\ \frac{60}{4} \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

x = 18.5

y = 15

Length = 18.5cm

Width = 15cm

Cramer's Rule

$x - y = 3.5$

$2x + 2y = 67$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 3.5 & -1 \\ 67 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 2 & 67 \end{bmatrix}$$

$|A| = (1)(2) - (-1)(2)$

$= 2 + 4$

$= 4$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 3.5 & -1 \\ 67 & 2 \end{vmatrix}}{4}$$

$$x = \frac{(3.5)(2) - (67)(-1)}{4}$$

$$x = \frac{7 + 67}{4}$$

$$x = \frac{74}{4}$$

$$y = \frac{|A_y|}{A}$$

$$y = \frac{\begin{vmatrix} 1 & 3.5 \\ 2 & 67 \end{vmatrix}}{4}$$

$$\frac{(1)(67)-(-2)(3.5)}{4} = \frac{67-7}{4}$$

$$\frac{60}{4} = 15$$

x = 18.5, y = 5

4: The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Sol. Let x , y be the angles of an isosceles triangle.

Then by given condition

$y = 2x - 160$ or $2x - 16 = y$

Or $2x - y = 16°$ ……… (i)

And $2x + y = 180°$ …… … (ii)

Solve: Sum of three angles of Δ is 1800

matrix from above equations.

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

A X = B

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$= (2)(1) - (2)(-1)$

$= 2 + 2 = 4 \neq 0$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

$X = A^{-1} \times B$

$X = \frac{1}{|A|} \text{Adj } A \times B$

$= \frac{1}{4}\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 16 \\ 180 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} (1)(16)+(1)(180) \\ (-2)(16)+(2)(180) \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 16+180 \\ -32+360 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 196 \\ 328 \end{bmatrix}$

$X = \begin{bmatrix} \frac{196}{4} \\ \frac{328}{4} \end{bmatrix}$

$X = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$

X = 49

Y = 82

So unknown angles are 49°, 49° and 82°

Cramer's Rule

$X = \frac{|A_x|}{|A|}$

$x = \frac{\begin{bmatrix} 16 & -1 \\ 180 & 1 \end{bmatrix}}{4} = \frac{(16)(1)-(180)(-1)}{4}$

$= \frac{16+180}{4} = \frac{196}{4} = 49^o$

$X = 49^o$

$y = \frac{|A_y|}{|A|}$

$y = \frac{\begin{bmatrix} 2 & 16 \\ 2 & 180 \end{bmatrix}}{4} = \frac{(2)(180)-(16)(2)}{4}$

$y = \frac{360^o - 32}{4} = \frac{328}{4} = 82^o$

So unknown angles are 49°, 49° and 82°

5: One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Sol. $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$

$A=\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2\times1-(-1\times1) = 2+1 = 3$

$X = A^{-1}B$

$X = \frac{AdjA}{A} \times B$

$X = \frac{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -12 \\ 90 \end{bmatrix}}{3}$

$X = \frac{\begin{bmatrix} -12 & +90 \\ 12 & +180 \end{bmatrix}}{3} = \frac{\begin{bmatrix} 78 \\ 192 \end{bmatrix}}{3}$

$= \begin{bmatrix} \frac{78}{3} \\ \frac{192}{3} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$

⇒ x = 26°, y = 64°

So one acute angle = 26°

Other acute angle = 64°

Cramer's rule

2x-y = –12

x + y = 90

$|A| = 3, A_x = \begin{bmatrix} -12 & -1 \\ 90 & 1 \end{bmatrix}$

$|A_x| = \begin{vmatrix} -12 & -1 \\ 90 & 1 \end{vmatrix}$

$|A_x| = -12+90 = 78$

$x = \frac{|A_x|}{|A|} = \frac{78}{3} = 26^o$

Now $A_y = \begin{vmatrix} 2 & -12 \\ 1 & 90 \end{vmatrix} = 2 \times 90 - (-12 \times 1)$

$= 180 + 12 = 192$

So $Ay = \dfrac{|A_y|}{|A|} = \dfrac{192}{3}$

$y = 64^o$

so one acute angle = 26^o

other acute angle = 64^o

6: Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after 4 ½ hours. Find the speed of each car.

Sol. Let the 1st speed of car = x km/hr

2nd speed of car = y km/hr

Difference between speeds = x – y = 6 . . . . . (i)

Time = $4\frac{1}{2}$ hours = $\frac{9}{2}$ + 123+(y) $\times \left(\frac{9}{2}\right)$

$\frac{9}{2} + \frac{9y}{2} = 600 - 123 = 477$

$\Rightarrow 9x + 9y = 954$

$9(x + y) = 954 \quad \Rightarrow (x + y) = \frac{954}{9} = 106$

x+y = 106. . . . . . (ii)

The matrix inverse method x – y = 6, x + y = 106

Sol.

By writting in matrix form of given equation

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

A X = B

$\Rightarrow \quad X = A^{-1} B$

And $A^{-1} = \frac{1}{|A|} AdjA$

$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1)(1) - (1)(-1)$

$|A| = 1 + 1 = 2$

As $|A| \neq 0$ so solution is possible.

Adj A$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$= A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 106 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} (1)\times(6)+(1)\times(106) \\ (-1)\times(6)+(1)\times(106) \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 6+106 \\ -6+106 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 112 \\ 100 \end{bmatrix} = \begin{bmatrix} \frac{112}{2} \\ \frac{100}{2} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$

Hence x = 56, y = 50

Cramer's rule

Sol. By writing in matrix form

A $= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, x$= \begin{bmatrix} x \\ y \end{bmatrix}$, B $= \begin{bmatrix} 6 \\ 106 \end{bmatrix}$

|A| $= \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$

= (1)(1) – (–1) (1)

= 1 + 1 = 2

As |A| $\neq 0$ so solution is possible.

Now $|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$

= (6)(1) –(–1) (106)

= 6+106=112

$A_y = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix}$

= (1)(106) –(1)(6)

= 106 –6 = 100

x $= \frac{|A_x|}{|A|} = \frac{112}{2}$ =56 and

x $= \frac{|A_y|}{|A|} = \frac{100}{2} = 100$

So, x = 56, y = 100

REVIEW EXERCISE 1

Select the correct answer in each of the following:

(i) The order of matrix [2 1] is

(a) 2– by– 1 (b) 1 –by– 2
(c) 1 –by– 1 (d) 2– by –2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called _______ matrix**

(a) zero (b) unit
(c) scalar (d) singular

(iii) Which is order of square matrix

(a) 2– by– 2 (b) – by– 2
(c) 2– by– 1 (d) 3– by– 2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is**

(a) 3 –by 2 (b) 2– by– 3
(c) 1– by –3 (d) 3– by– 1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of [x y] $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is**

(a) [2x + y] (b) [x – 2y]
(c) [2x – y] (d) [x + 2y]

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$ **= 0, then x is equal to**

(a) 9 (b) –6
(c) 6 (d) –9

(viii) If X + $\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **then x is equal to**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWERS

(i)	b	**(ii)**	c	**(iii)**	a	**(iv)**	b
(v)	a	**(vi)**	c	**(vii)**	a	**(viii)**	d

Q.2: Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called _________ matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called _________ matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is ______

(iv) Matrix multiplication in general AB_________BA.

(v) Matrix A + B may be find if order of A and B is ______________

(vi) A matrix is called ________ matrix if number of rows and columns are equal.

ANSWERS

(i)	null	**(ii)**	Unit	**(iii)**	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$
(iv)	≠	**(v)**	same	**(vi)**	square

Q.3: If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find a and b.**

According to define of equal matrices.

Sol. $a + 3 = -3$

$a = -3 - 3$

$a = -6$

$b - 1 = 2$

$b = 2 + 1 \Rightarrow$ **b = 3**

Q.4: If A = $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$**, B =** $\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$ **then find the following:**

(i) 2A + 3B **(ii) –3A + 2B**

(iii) –3(A + 2B) **(iv)** $\frac{2}{3}$ **(2A – 3B)**

(i) 2A + 3B

Sol. Find 2A + 3B

$= 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$

$= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$

(ii) –3A + 2B

Sol. $= -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{4} & \mathbf{-17} \\ \mathbf{-7} & \mathbf{-2} \end{bmatrix}$

(iii) –3(A + 2B)

Sol. $-3(A + 2B) = -3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$

$= -3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}\right)$

$= -3\left(\begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix}\right)$

$= -3\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$

$\mathbf{-3(A + 2B)} = \begin{bmatrix} \mathbf{-36} & \mathbf{15} \\ \mathbf{9} & \mathbf{6} \end{bmatrix}$

(iv) $\mathbf{\frac{2}{3}(2A - 3B)}$

Sol. $= \frac{2}{3}\left(2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$

$= \frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$

$= \frac{2}{3}\left(\begin{bmatrix} 4-15 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}\right) = \frac{2}{3}\begin{bmatrix} 11 & 18 \\ 8 & 3 \end{bmatrix}$

$= \begin{bmatrix} (\frac{2}{3})(-11) & (\frac{2}{3})(18) \\ (\frac{2}{3})(8) & (\frac{2}{3})(3) \end{bmatrix}$

$= \begin{bmatrix} \mathbf{\frac{-22}{3}} & \mathbf{12} \\ \mathbf{\frac{16}{3}} & \mathbf{2} \end{bmatrix}$.

Q.5: Find the value of X, if $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{-3} \end{bmatrix}$ **+ X =** $\begin{bmatrix} \mathbf{4} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{-2} \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$

$X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$

$X = \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2+3 \end{bmatrix}$

$\mathbf{X} = \begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{1} \end{bmatrix}$ Ans.

Q.6: If A = $\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{2} & \mathbf{-3} \end{bmatrix}$**, B =** $\begin{bmatrix} \mathbf{-3} & \mathbf{4} \\ \mathbf{5} & \mathbf{-2} \end{bmatrix}$ **then prove that:**

(i) AB ≠ BA (ii) A(BC) = (AB)C

(i) AB ≠ BA

Sol. $AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} (0)(-3)+(1)(5) & (0)(4)+(1)(-2) \\ (2)(-3)+(-3)(5) & (2)(4)+(-3)(-2) \end{bmatrix}$

$AB = \begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix}$ . . . . . (i)

$AB = \begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix}$

$BA = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} (-3)(0)+(4)(2) & (-3)(1)+(4)(-3) \\ (5)(0)+(-2)(2) & (5)(1)+(-2)(-3) \end{bmatrix}$

$= \begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix}$

$BA = \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix}$ . . . .(ii)

From (i) and (ii) Its proved that AB # BA

Q.7: If A = $\begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix}$ **and B =** $\begin{bmatrix} \mathbf{2} & \mathbf{4} \\ \mathbf{-3} & \mathbf{-5} \end{bmatrix}$ **then verify that:** **(i)** $\mathbf{(AB)^t = B^tA^t}$

(ii) $\mathbf{(AB)^{-1} = B^{-1}A^{-1}}$

(i) $\mathbf{(AB)^t = B^tA^t}$

Sol. $(AB)^t$

L.H.S.

$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} (3)(2)+(2)(-3) & (3)(4)+(2)(-5) \\ (1)(2)+(-1)(-3) & (1)(4)+(-1)(-5) \end{bmatrix}$

$AB = \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$

Taking transport

$(AB)^t = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t$

$= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix}$

R.H.S B^tA^t

$B^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$, $A^t = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$,

$B^tA^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} (2)(3)+(-3)(2) & (2)(1)+(-3)(-1) \\ (4)(3)+(-5)(2) & (4)(1)+(-5)(-1) \end{bmatrix}$

$= \begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix}$

$B^tA^t = \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix}$

From (i) and (ii) its proved that $(AB)^t = B^tA^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Sol. $(AB)^{-1}$

$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} (3)(2)+(2)(-3) & (3)(4)+(2)(-5) \\ (1)(2)+(-1)(-3) & (1)(4)+(-1)(-5) \end{bmatrix}$

$= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

We already have

$AB = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$

$|AB| = \begin{vmatrix} 0 & 2 \\ 5 & 9 \end{vmatrix} = (0)(9) - (2)(5)$

$|AB| = 0 - 10 = -10 \neq 0$ Non-singular matrix so solution possible.

$\text{Adj } AB = \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|} \text{Adj } AB$

$= -\frac{1}{10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix} x$

As $B^{-1} = \frac{1}{|B|} \text{Adj } B$

$|B| = \begin{vmatrix} 2 & 4 \\ -3 & -5 \end{vmatrix} = (2)(-5) - (-3)(4)$

$= -10 + 12 = 2$

$B^{-1} = \frac{1}{2}\begin{bmatrix} -5 & -4 \\ +3 & 2 \end{bmatrix}$ …..(i)

As $A^{-1} = \frac{1}{|A|} \text{Adj } A$

$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = (3)(-1) - (1)(2) = -3 - 2$

$= -5$

$A^{-1} = \frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$ x

Multiply (i) × (ii)

$B^{-1} \times A^{-1} = \frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \times \frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{2} \times \frac{1}{-5}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

$= \frac{-1}{10}\begin{bmatrix} (-5)(-1)+(-1)(-4) & (-5)(-2)+(-4)(3) \\ (3)(-1)+(2)(-1) & (3)(-2)+(2)(3) \end{bmatrix}$

$= \frac{-1}{10}\begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}$

$= \frac{-1}{10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix} = y$

See equation x and y $(AB)^{-1} = B^{-1} \times A^{-1}$

IMPORTANT KEY POINTS OF CHAPTER

- A rectangular array of real numbers enclosed with brackets is said to form a matrix.
- A matrix A is called rectangular, if the number of rows and number of columns of A are not equal.
- A matrix A is called a square matrix, if the number of rows of A is equal to the number of columns.
- A matrix A is called a row matrix, if A has only one row.
- A matrix A is called a column matrix, if A has only one column.
- A matrix A is called a null or zero matrix, if each of its entry is 0.
- Let A be a matrix. The matrix A^t is a new matrix which is called transpose of matrix A and is obtained by interchanging rows of A into its respective columns (or columns into respective rows).
- A square matrix A is called symmetric, if $A^t = A$.

- Let A be a matrix. Then its negative, -A, is obtained by changing
- The signs of all the non - zero entries of A.
- A square matrix M is said to be skew symmetric, if $M^t = -M$,
- A square matrix M is called a diagonal matrix, if atleast any one of entry of its diagonal is not zero and all non diagonal entries are zero.
- A scalar matrix is called identity matrix, if all diagonal entries are 1.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is called 3-by-3 identity matrix,

- Let A be a matrix of order 2-by-3. Then a matrix B of same order is said to be an additive identity of matrix A, if B + A = A = A + B
- Let A be a matrix. A matrix B is defined as an additive inverse of A, if
  B + A = O = A + B
- Let A be a matrix. Another matrix B is called the identity matrix of A under multiplication, if B × A = A = A × B.
- Let M = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by-2 matrix. A real number is called determinant of M, denoted by det M such that

  det M = $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ = ad – bc = λ
- A square matrix M is called singular, if the determinant of M is equal to zero.
- A square matrix M is called non-singular, if the determinant of M is not equal to zero.
- For a matrix M = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adjoint of M is defined by

  Adj M = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- Let M be a square matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

  $M^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where,

  det M = $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ = ad – bc = λ ≠ 0
- Any two matrices A and B are called equal, if
  order of A = order of B (ii) corresponding entries are equal
- Any two matrices M and N are said to be conformable for addition, if order of M = order of N.
- The following laws of addition hold M + N = N +M (Commutative)
  (M + N) + T = M + (N + T) (Associative)
- The matrices M and N are conformable for multiplication to obtain MN if the number of columns of M = number of rows of N, where

(i) (MN) ≠ NM, in general
(ii) (MN)T = M(NT) (Associative law)
(iii) M(N + T) = MN + MT
(iv) (N + T)M = NM + T
} (Distributive laws)

- Law of transpose of product $(AB)^t = B^t A^t$
  $AA^{-1} = I = A^{-1}A$
- The solution of a linear system of equations,
  $ax + by = m$
  $cx + dy = n$
  by expressing in matrix form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$ is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}\begin{bmatrix} m \\ n \end{bmatrix}$ if the coefficient matrix is non-singular.
- By using the Cramer's rule the determinental form of the solution is

$$x = \frac{\begin{vmatrix} m & b \\ n & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \text{ and } y = \frac{\begin{vmatrix} a & m \\ c & n \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Where $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

ADDITIONAL MCQ'S

1. Adjoint of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is ……

(a) $\begin{bmatrix} a & -b \\ c & d \end{bmatrix}$ (b) $\begin{bmatrix} a & b \\ c & -d \end{bmatrix}$

(c) $\begin{bmatrix} d & -b \\ c & a \end{bmatrix}$ (d) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

2. The idea of matrices was given by ……

(a) Aurthur Cayley (b) Briggs

(c) Al-Khwarzmi (d) Thomas Harriot

3. A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then |A| = ……

(a) ab-cd (b) ac-bd

(c) bc-ad (d) ad-bc

4. Aurther Cayley introduced theory of matrices in ……

(a) 1854 (b) 1856

(c) 1858 (d) 1860

5. For …… values of x $\begin{bmatrix} 3 & -6 \\ 2 & x \end{bmatrix}$ will be a singular matrix.

(a) –3 (b) –4

(c) 3 (d) 4

6. Product of [1 2] $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

(a) [–13] (b) [–3]

(c) [3] (d) [13]

7. If $\begin{bmatrix} a+3 & 4 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 0 \end{bmatrix}$ value of 'a' will be.

(a) 6 (b) 3

(c) –3 (d) –6

8. The order of matrix $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$.

(a) 1–by–3 (b) 3–by–1

(c) 3–by–3 (d) 2–by–2

9. A square matrix M is called to be skew symmetric,

(a) $M^t = \overline{M}$ (b) $M^t = \frac{1}{M}$

(c) $M^t = -M$ (d) $M^t = M$

10. A square matrix M is called to be symmetric matrix if:

(a) $M^t = \overline{M}$ (b) $M^t = \frac{1}{M}$

(c) $M^t = -M$ (d) $M^t = M$

ANSWERS

(i)	c	(ii)	a	(iii)	d	(iv)	c
(v)	b	(vi)	b	(vii)	d	(viii)	b
(ix)	c	(x)	d				